Fostering Collaboration

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To compete or not?

Orgs use internal competition when best approach unclear.

Maintaining competing approaches provides option value.

But competition yields wasteful duplication of effort.

Intra-party political competition

US presidential primaries

Primaries are useful to parties in identifying candidates with higher valence, something that is hard to know ex-ante.

Carey and Polga-Hecimovich (2006)

[In] many plausible scenarios the strategic advantage arising from the primary electorate's ability to select a high-quality nominee …outweighs the strategic disadvantage that the primary pulls the party's nominee away from the center of the general electorate.

Adams and Merrill III (2008)

Internal competition in firms

Telstar Communications

IT infrastructure company…Telstar…found itself running two competing middleware technology platforms, [AX and EX]. The competition between these two platforms came to a head when the new business unit had to choose one…[T]op-level executives were brought in and they decided to choose in favor of EX…However, they also placed the two warring teams in the same group in order [to] build a common platform for future use.

"Strategies for Managing Internal Competition"

Birkinshaw (2001)

Common elements

Organization has to make decision.

Uncertainty to be resolved on which option is best.

Members decide which option to develop.

Members' interests are opposed.

Tension between developing options and choosing right one.

What is the best selection rule?

What we do in this paper

We study an optimal contracting setting:

- Principal must pick one of the agents' projects at the deadline; each agent wants his project chosen
- At every instant, agents allocates a unit of (fixed total) effort between working on his own project and on other's.
- Project evolution depends on effort and exogenous Brownian motion.

What we do in this paper

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- At every instant, agents allocates a unit of (fixed total) effort between working on his own project and on other's.
- Project evolution depends on effort and exogenous Brownian motion.

And we find:

- Unique principal-optimal selection rule: time-dependent, decreasing lead threshold z^*; a project is chosen at first time t when it overtakes the other by z_t^*.
- Initially, work on own project; then collaborate on winner.

Related literature

A brief overview

Organizations as political coalitions

- March (1962); Cyert & March (1963),...

  This paper: Fostering collaboration given empire motives.

Strategic experimentation

- Bonatti & Hörner (2011); Bonatti & Rantakari (2016),...

  This paper: No free-riding. Exploration-exploitation trade-off endogenous from agency problem.

Dynamic contracting without transfers

- Guo & Hörner (2020); McClellan (2021),...

  This paper: Martingale methods, weak solutions.

Model

Competition in organizations

- Two workers $i \in \{1, -1\}$; each "owns" a project i.
- Time runs $t \in [0, T]$, worker i chooses $a_t^i \in [0, 1]$.
- Project i runs according to

$$dX_t^i = \left\{\beta + \mu\left[a_t^i + (1 - a_t^{-i})\right]\right\} dt + \sigma \, dB_t^i,$$

  where $\mu, \sigma > 0$ and $B^1 \perp B^{-1}$ are standard Brownian.
- Firm chooses $y \in \{1, -1\}$ given $\{X_t^1, X_t^{-1}\}_{t \in [0,T]}$ at time T.
- Payoff X_T^y to firm, iy to worker i.
- X_t^i publicly observed, effort allocation choices not.

Key modeling assumptions

No margin for total effort.

Workers are purely 'empire' concerned.

Effort is purely instrumental.

Moral hazard: only project evolution public.

No transfers.

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Linear profits, independent increments; two agents.

Benchmarks

First-best solution

Firm chooses best project ex-post.

Each worker works on the current leader.

Lesson: Want to be adaptive and collaborative.

Ex-post optimal project choice

Firm chooses best project ex-post.

Each worker works on own project.

Lesson: Being adaptive induces competition.

Can commitment be useful?

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1. Principal's Pet Project: Principal commits to picking her pet project i, and has agents collaborate on the pet project.

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1. Principal's Pet Project:

2. Unassailable Lead:

Can commitment be useful?

1. Principal's Pet Project:

2. Unassailable Lead: Agents start out competing. Principal commits to picking a project irreversibly if it is the first to take on a lead of at least L. Agents then collaborate on this chosen project.

Can commitment be useful?

1. Principal's Pet Project:

2. Unassailable Lead:

3. Early Lead Advantage:

Can commitment be useful?

1. Principal's Pet Project:

2. Unassailable Lead:

3. Early Lead Advantage:
   - If i is first to take a lead of L, choose i with prob. $\frac{3}{4}$ at T.
   - If subsequently, $-i$ catches up, and $-i$ is ahead at the deadline, then choose i with prob. $\frac{1}{2}$.
   - But, if i is again ahead at T, then chose i with prob. 1.
   - Agents start out competing. When an early leader emerges, both agents collaborate on early leader, and start competing again if/when the early lead disappears.

Optimal
Selection Rule

Punchline in brief

Optimal policy follows simple form:

First compete, then collaborate.

1. Firm sets a decreasing lead threshold $(\bar{z}_t)_t$.
2. Workers compete until a winner i has $i\Delta X_t \geq \bar{z}_t$.
3. Workers collaborate on the winner until T.

PRINCIPLES GUIDING THE CHARACTERIZATION

1. Collaboration on the current “favorite” project.
2. Decide quickly.
3. Ignore aggregate performance.
4. First compete, then collaborate on the winner.
5. The winner is the first to take a large enough lead.

Convenient normalizations

WLOG, have $\sigma = \mu = 1$ and $\beta + \mu = X_0^1 + X_0^{-1} = 0$.

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Think of state variables as $\Sigma X_t = X_t^1 + X_t^{-1}$ and $\Delta X_t = X_t^1 - X_t^{-1}$.

$$\implies \Sigma X_t = \Sigma B_t \text{ and } \mathrm{d}\Delta X_t = 2\Delta a_t \, \mathrm{d}t + \mathrm{d}\Delta B_t,$$

and firm's objective is

$$\mathbb{E}[X_T^y] = \mathbb{E}\left[y\left(\frac{1}{2}\Delta X_T + \frac{1}{2}\Sigma X_T\right)\right] = \frac{1}{2}\mathbb{E}\left[y\Delta X_T\right].$$

Recursive approach

Let $q_t := \mathbb{E}_t y$, which serves as promised utility for workers.

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Firm chooses (a_t^1, a_t^{-1}) and evolution of q_t to maximize

$$\tfrac{1}{2}\mathbb{E}[q_T \Delta X_T]$$

subject to worker IC and q being a martingale.

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MRT $\rightsquigarrow$ process $\{c_t^\Delta, c_t^\Sigma\}_t$ such that

$$\mathrm{d}q_t = c_t^\Delta \left[\mathrm{d}\Delta X_t - 2\Delta a_t \,\mathrm{d}t\right] + c_t^\Sigma \,\mathrm{d}\Sigma X_t.$$

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Now, IC can be written as $a_t^i = \begin{cases} 1 & : \; c_t^\Delta > 0 \\ 0 & : \; c_t^\Delta < 0. \end{cases}$

Where we are

1. Collaboration on the current "favorite" project.
    - When collaboration happens, it happens on whichever project the principal is currently more likely to choose—which is endogenous.

2. Decide quickly.

3. Ignore aggregate performance.

4. First compete, then collaborate on the winner.

5. The winner is the first to take a large enough lead.

Our firm's program

Claim: The firm solves

$$\max_{q,\, c^{\Delta},\, c^{\Sigma},\, a} \left\{ \underbrace{\frac{1}{2} q_0 \Delta X_0}_{\text{ex-ante}} + \mathbb{E} \int_0^T \left[\underbrace{(\Delta a_t) q_t}_{\text{collaboration}} + \underbrace{c_t^{\Delta}}_{\text{adaptivity}} \right] \mathrm{d}t \right\}$$

$$\text{s.t.} \quad \mathrm{d}q_t = c_t^{\Delta}\, \mathrm{d}\Delta B_t + c_t^{\Sigma}\, \mathrm{d}\Sigma B_t, \quad -1 \le q \le 1;$$

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Relaxed program: Let firm choose $(\Delta B, \Sigma B)$ too, subject to law.

Where to collaborate?

$$\max_{q,\, c^{\Delta},\, c^{\Sigma},\, a} \quad \left\{ \frac{1}{2} q_0 \Delta X_0 + \mathbb{E} \int_0^T \left[(\Delta a_t) q_t \;\; + c_t^{\Delta} \right] \mathrm{d}t \right\}$$

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Modify a_t when $c_t^{\Delta} = 0$, while keeping $(q, c^{\Delta}, c^{\Sigma})$ fixed:

- Preserves constraints.
- Changes $(\Delta a_t) q_t$ part of objective and nothing else.

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- Preserves constraints.
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So have agents work on current favorite: $(\Delta a_t) q_t = |q_t| \mathbf{1}_{c_t^{\Delta} = 0}$.

Where we are

1. Collaboration on the current "favorite" project.

2. Decide quickly.
   - Until either the decision is made ($|q_t| = 1$) or the deadline arrives ($t = T$), the principal resolves uncertainty about project choice somewhat quickly: $\left\|(c_t^{\Delta}, c_t^{\Sigma})\right\|_2 \geq 1$.

3. Ignore aggregate performance.

4. First compete, then collaborate on the winner.

5. The winner is the first to take a large enough lead.

Why decide quickly?

Firm solves

$$\max_{q,\, c^{\Delta},\, c^{\Sigma}} \quad \left\{ \frac{1}{2} q_0 \Delta X_0 + \mathbb{E} \int_0^T \left[|q_t| \mathbf{1}_{c_t^{\Delta}=0} + c_t^{\Delta} \right] \mathrm{d}t \right\}$$

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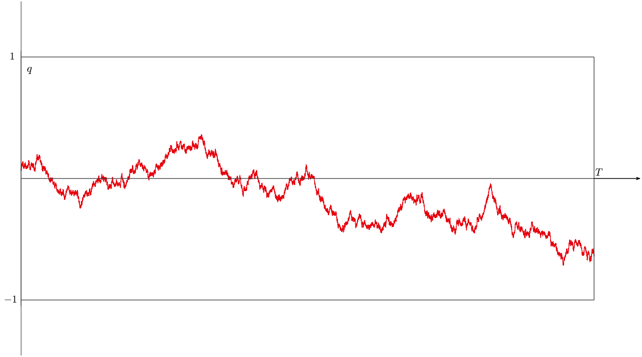
But consider "flow payoff" term $f_t = |q_t| \mathbf{1}_{c_t^\Delta = 0} + c_t^\Delta$:

- When $|q_t| < 1$ and $\left\| (c_t^\Delta, c_t^\Sigma) \right\| < 1$, have $|f_t| < 1$.
- Can always guarantee flow $f = 1$ from t onward:

$$(c^\Delta, c^\Sigma) = (1, 0) \text{ until } q = 1.$$

CONSTRUCTION: TIME CHANGE



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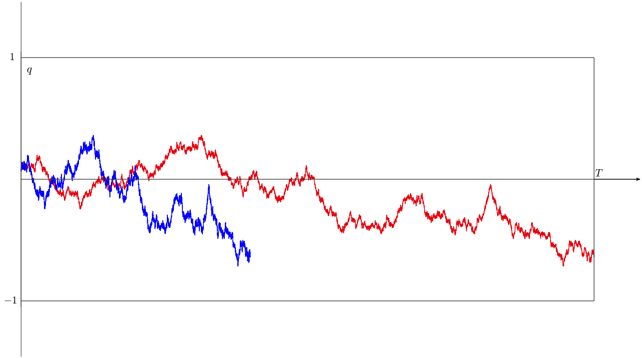

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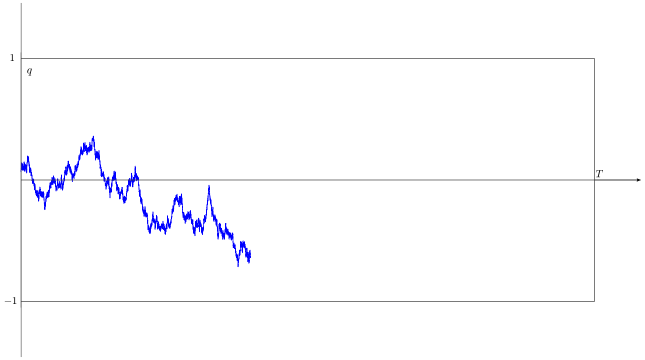

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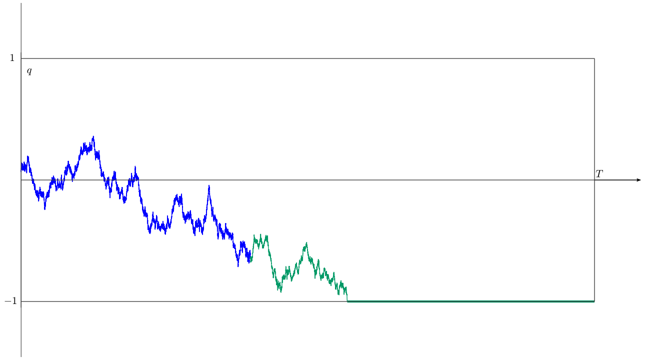

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Where we are

1. Collaboration on the current "favorite" project.
2. Decide quickly.
3. Ignore aggregate performance.
   - Decide based on relative performance of projects (ΔX), but do not respond to aggregate performance (ΣX).
4. First compete, then collaborate on the winner.
5. The winner is the first to take a large enough lead.

Why ignore aggregate shocks?

Back to "flow payoff" term $f_t = |q_t| \mathbf{1}_{c_t^{\Delta}=0} + c_t^{\Delta}$.

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- Consider replacing (c_t^Δ, c_t^Σ) with $(c_t, 0)$.

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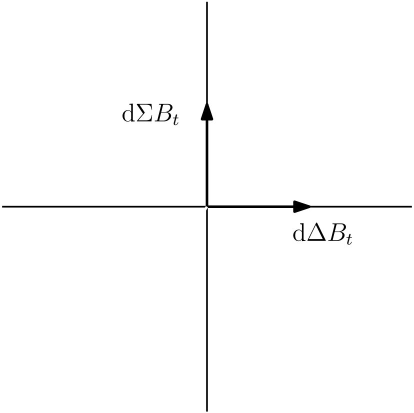
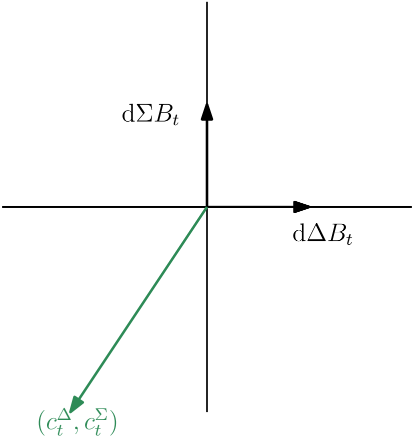
- Consider replacing (c_t^Δ, c_t^Σ) with $(c_t, 0)$.
- Looking separately at $c_t^\Delta \neq 0$ and $c_t^\Delta = 0$ case, see raises f_t.

CONSTRUCTION: ROTATION



$$\frac{1}{2} q_0 \Delta X_0 + \mathbb{E} \int_0^T \left[|q_t| \mathbf{1}_{c_t^\Delta = 0} + c_t^\Delta \right] \mathrm{d}t$$

Construction: rotation



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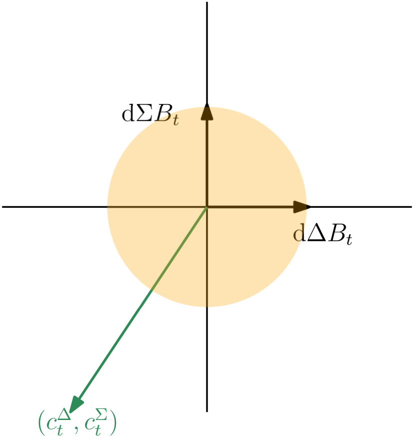

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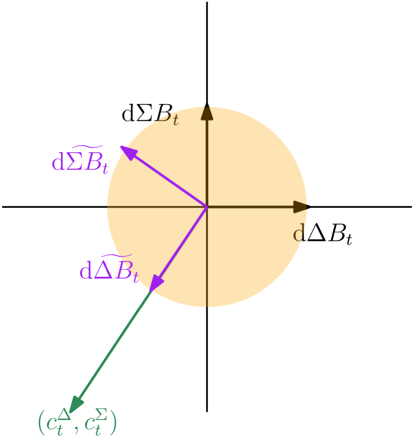

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Where we are

1. Collaboration on the current "favorite" project.
2. Decide quickly.
3. Ignore aggregate performance.
4. First compete, then collaborate on the winner.
   - Works on own project until some stopping time τ, and then work on whichever project has a higher state as of τ.
5. The winner is the first to take a large enough lead.

First compete, then collaborate—on winner

Let $\tau = T \wedge \inf\{t \in [0,T] : \ |q_t| = 1\}$. By above:

First compete, then collaborate—on winner

Let $\tau = T \wedge \inf\{t \in [0, T] : |q_t| = 1\}$. By above:

- All work on own projects before τ (since $c_t^{\Delta} > 0$).
- Project choice independent of shocks after τ.
- Collaboration on chosen project from τ onward.

First compete, then collaborate—on winner

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- All work on own projects before τ (since $c_t^{\Delta} > 0$).
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Optimize over τ and project s.t. these constraints (ignoring IC):

FIRST COMPETE, THEN COLLABORATE—ON WINNER

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- All work on own projects before τ (since $c_t^{\Delta} > 0$).
- Project choice independent of shocks after τ.
- Collaboration on chosen project from τ onward.

Optimize over τ and project s.t. these constraints (ignoring IC):

- Will choose winner: higher X_τ^i.

Where we are

1. Collaboration on the current "favorite" project.
2. Decide quickly.
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4. First compete, then collaborate on the winner.
5. The winner is the first to take a large enough lead.
   - Set lead threshold that decreases as deadline approaches, and choose first project that takes such a lead.

Lower standards as option value declines

$$
\begin{array}{ll}
\sup_{\tau} & \mathbb{E}\left[\frac{1}{2}\,|\Delta X_{\tau}| \;+\; (T-\tau)\right] \\
\text{s.t.} & \tau \text{ is a stopping time},\; 0 \le \tau \le T.
\end{array}
$$

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This is an “easy” optimal stopping problem:

- Watch Brownian motion $\frac{\Delta X}{2}$ evolve, for a flow cost 1.
- Decide when to stop, and collect stopping value $\left|\frac{\Delta X_{\tau}}{2}\right|$.
- Can stop anytime before a *deadline* T.

Main theorem

Theorem: There is a lead threshold $(\bar{z}_t)_{t\in[0,T)}$ such that, letting

$$\tau := T \wedge \inf\{t \in [0,T] : |\Delta X_t| \geq \bar{z}_t\},$$

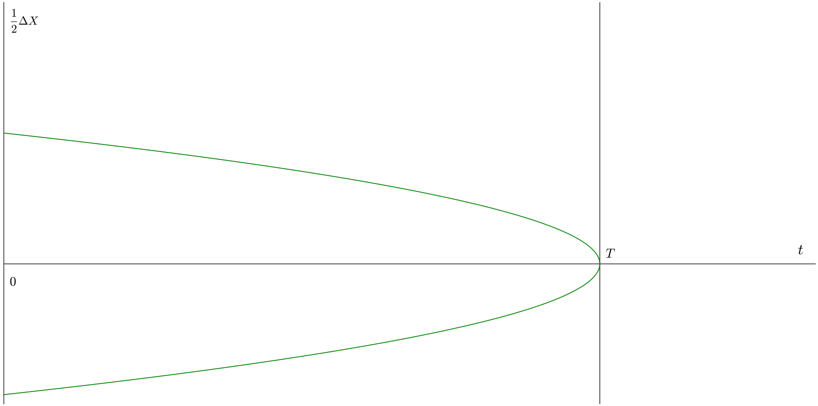
the following policy is uniquely optimal:

- *Compete* until τ;
- *Collaborate* on the τ-leader after τ.

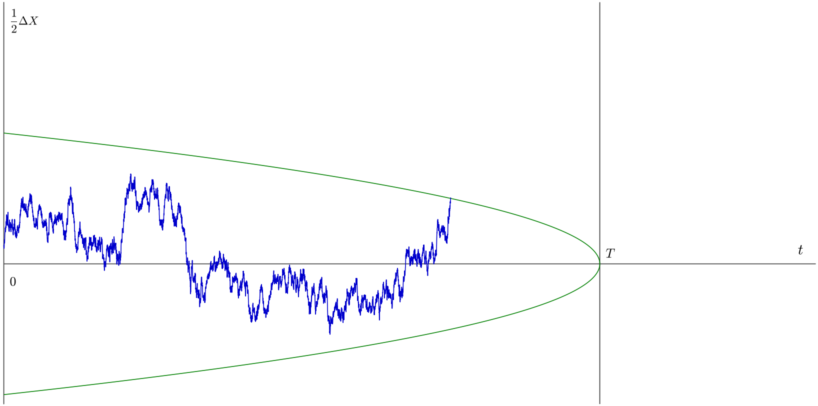
Moreover, $\bar{z}_t$ is bounded and decreasing in t, strictly positive, continuous, and converges to zero as $t \to T$.

OPTIMAL CONTRACT

Optimal contract



Optimal contract



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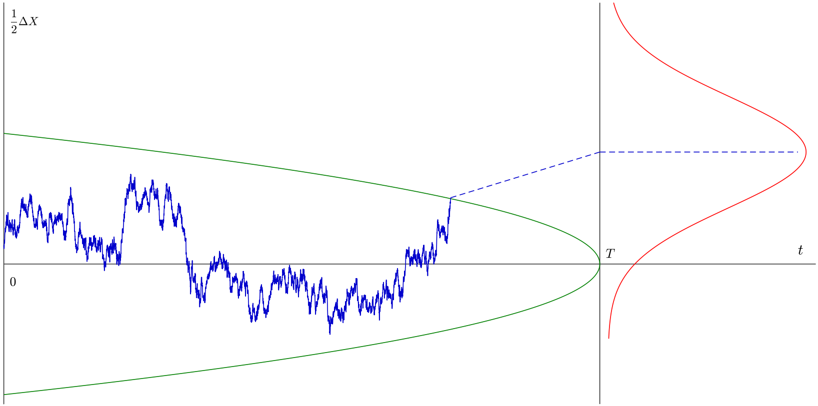
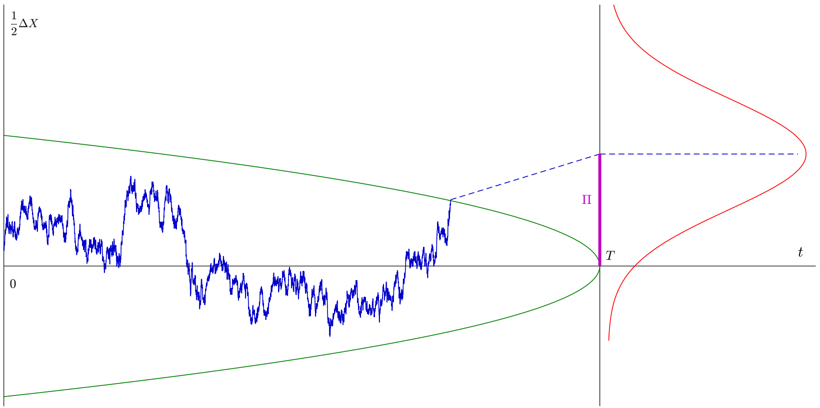


Optimal contract



Discussion

Collaboration duration

For long projects, most development time is collaborative.

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Collaboration begins at some $\tau < T$.

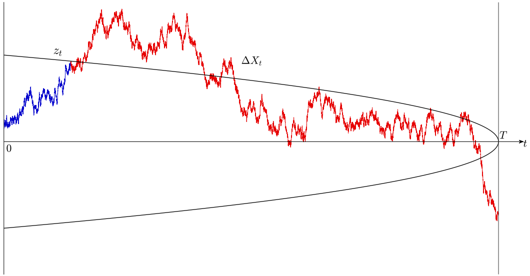
- Threshold collapses as the deadline approaches.

Even as $T \to \infty$, competition duration is FOSD bounded.

- Comparing to no-deadline stopping problem

Ex-post inefficiency

Arbitrarily large ex-post inefficiencies happen on path.



- Collaboration increases the value of the chosen project.
- But, more noise in quality of decision making.

Canceling projects before the deadline

Suppose firm can irreversibly scrap one project at any time.

- Equivalent contracting problem.

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- Firm scraps a project iff it's losing by at least $\bar{z}_t$.
- Agents work on own project whenever allowed.

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- Firm scraps a project iff it's losing by at least $\bar{z}_t$.
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This profile is strictly sequentially rational.

- Firm needs little commitment power.

Indifference during collaboration

While collaborating, agents are indifferent between competing and collaborating. *Is this troubling?*

- With no transfers, no observability, and directly opposed agents, the principal has few instruments at her disposal.

  Some collaboration is possible and optimal despite this.

- Is irreversible choice necessary to foster collaboration?

  No. But it is a feature of *second-best* solution.

- Our model = limiting case with large empire motives.

  Small monetary budget restores strict incentives.

Wrapping Up

Some highlights

Organizations want to balance three things:

1. Good ex-ante project choices;
2. Efficient collaboration.
3. Adaptability to changing circumstances.

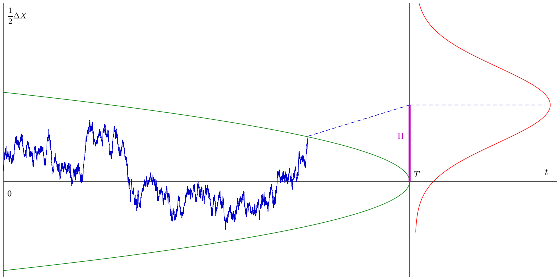
Order matters: first compete, then collaborate.

- Backload collaboration to make it more useful.

Unboundedly large inefficiencies are efficient.

Shutting down projects implements commitment solution.

Thanks!



Decomposing the firm's objective

▸ ...

Lemma: Firm expected profit is

$$\frac{1}{2}\underbrace{q_0 \Delta X_0}_{\text{ex-ante}} + \mathbb{E}\int_0^T \left\{ \underbrace{(\Delta a_t) q_t}_{\text{collaboration}} \, \mathrm{d}t + \frac{1}{2} \underbrace{\mathbb{E}_t \left[(\mathrm{d}q_t)(\mathrm{d}\Delta X_t) \right]}_{\text{adaptivity}} \right\}$$

DECOMPOSING THE FIRM'S OBJECTIVE

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"Proof": Given normalizations, have

$$\Pi_0 = \mathbb{E}\left[\tfrac{1}{2}q_T\Delta X_T\right] = \tfrac{1}{2}q_0\Delta X_0 + \mathbb{E}\int_0^T \tfrac{1}{2}\mathbb{E}_t \mathrm{d}\left(q_t\Delta X_t\right).$$

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$$\begin{aligned} \text{But} \qquad & \frac{1}{2}\mathbb{E}_t \mathrm{d}\left[q_t \Delta X_t\right] \\ = \quad & \frac{1}{2}\mathbb{E}_t \left\{(\Delta X_t)\,\mathrm{d}q_t + q_t\,\mathrm{d}\Delta X_t + (\mathrm{d}q_t)\,\mathrm{d}(\Delta X_t)\right\} \\ = \quad & 0 + (\Delta a_t) q_t\,\mathrm{d}t + \frac{1}{2}\mathbb{E}_t\left[(\mathrm{d}q_t)\,\mathrm{d}\Delta B_t\right]. \end{aligned}$$