Repeated Delegation

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Delegation Everywhere

Those with authority may differ from those with information.

- State delegates infrastructure spending to municipalities.
- Conglomerate delegates investment choice to subsidiaries.
- NGO delegates funding allocation to local governments.
- University delegates academic hiring to departments.

Limited *liability*, limited *information*, limited *commitment*.

Ongoing Delegation

Repeated game: *principal* has authority over a decision, *agent* holds relevant information.

Each period, *principal* and *agent* face a new project.

P wants projects that A values, but also bears a cost.

$\implies$ P wants only good projects; A wants all projects.

P needs A: can't assess projects without A's expertise.

Questions & Answers

What we do

1. How productive can the relationship be?
2. How does the relationship evolve over time?
3. How should delegation be implemented?

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What we do

1. How productive can the relationship be?

   Full characterization of equilibrium payoff set (at fixed δ).

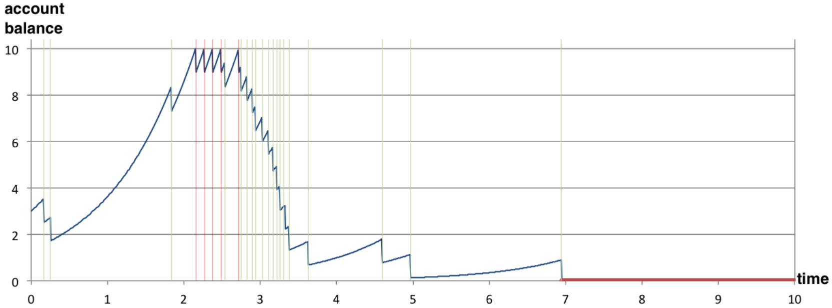
2. How does the relationship evolve over time?

   Frontloaded reward: first overfund, then underfund.

3. How should delegation be implemented?

   Budgeting: expense account, market interest rate, cap.

Pareto Optimal Equilibrium: Preview



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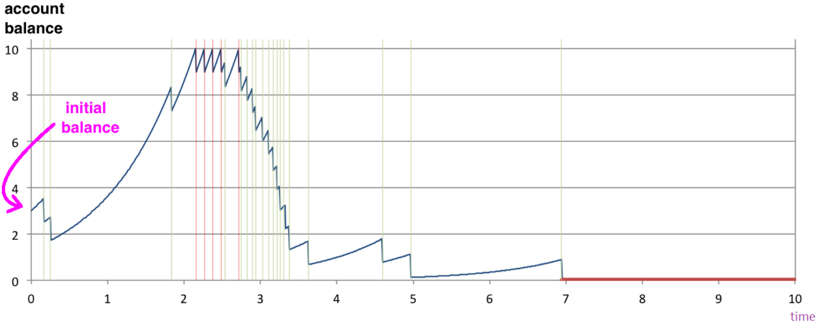

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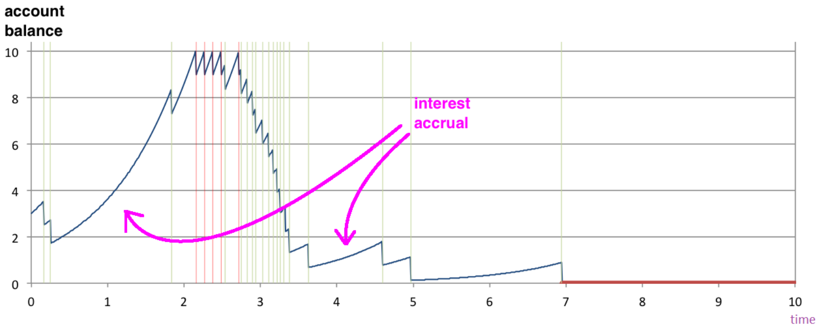

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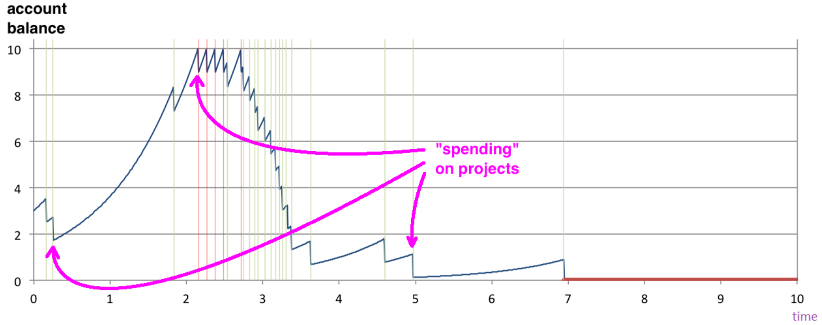

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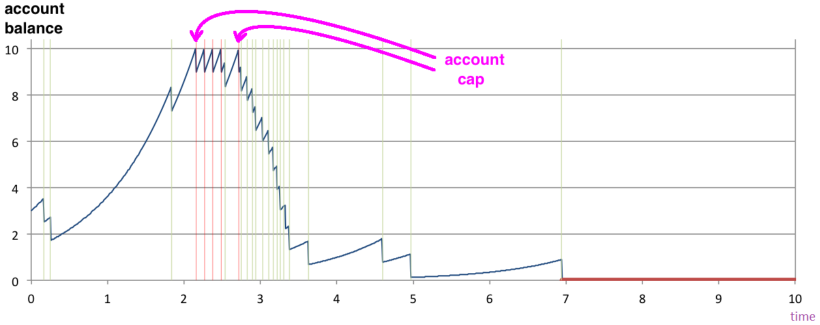

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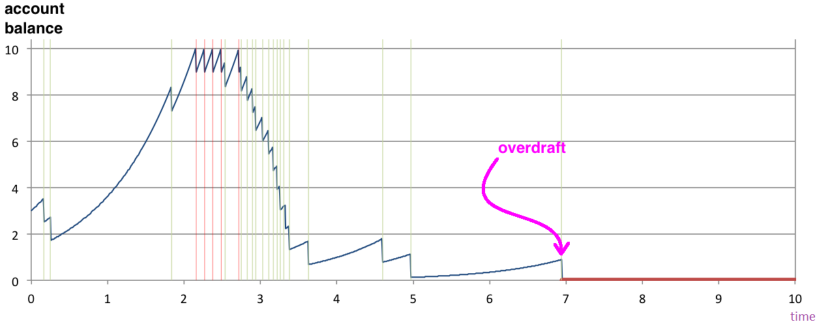

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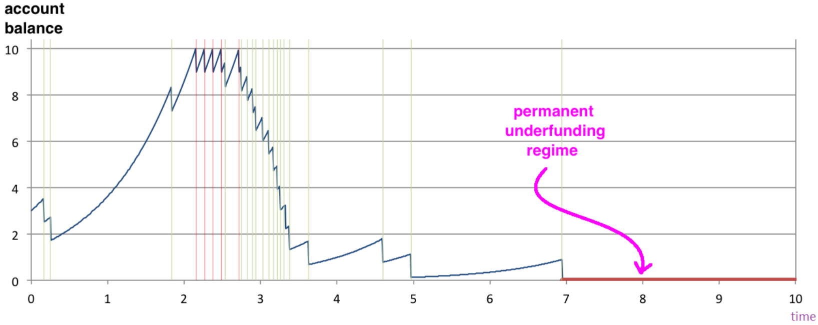

Related Literature

Dynamic Contracting under Limited Liability

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- Thomas & Worrall ('90); Guo & Hörner ('15)
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Today:

no commitment + limited info $\implies$ long-run punishment

Related Literature, Ctd.

Delegation:

- Holmström ('84); Armstrong & Vickers ('10); Frankel ('14); Ambrus & Egorov ('15)

Linked decisions:

- Casella ('05); Jackson & Sonnenschein ('07); Frankel ('16)

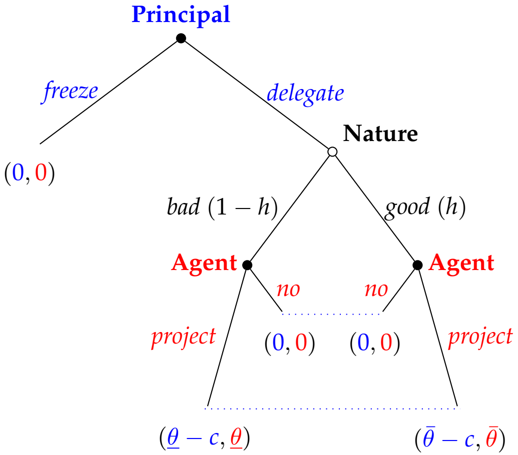
Dynamic corporate finance:

- Clementi & Hopenhayn ('06), Biais et al. ('10)

Relationship-building with private information:

- Hauser & Hopenhayn ('08); Li & Matouschek ('13)

The Stage Game



Project types $\theta \in \{\bar{\theta}, \underline{\theta}\}$ satisfy: $0 < \underline{\theta} < \mathbb{E}[\theta] < c < \bar{\theta}$

Our Repeated Delegation Game

- Discrete time, i.i.d. types $(\theta_k)_k$, discount factor $\delta > 0$.

$$\text{Agent value } v = (1-\delta)\sum_{k=0}^{\infty}\delta^k \mathbf{1}_{\{\text{project in period } k\}}\theta_k$$

- Actions perfectly observed, but principal never sees θ_k.
- Public Perfect Equilibrium.
- Nash reversion $\implies$ only need to think about on-path.

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$$\text{Principal profit } \pi = (1-\delta)\sum_{k=0}^{\infty}\delta^k \mathbf{1}_{\{\text{project in period } k\}}(\theta_k - c)$$

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Where we're headed

1. Understanding dynamic delegation
2. Characterizing the equilibrium value set
3. Long-run dynamics
4. Implementing Pareto optimal equilibria

1. **Understanding dynamic delegation**
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A Non-trivial Equilibrium

One project, any time

(P) Delegate until a project is adopted.

After first project, freeze forever.

(A) Adopt only good projects.

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Fact: The above is an equilibrium if and only if

$$\delta\omega \geq (1-\delta)\underline{\theta},$$

where $\omega = \mathbb{E}[\theta] - \underline{\theta}$, the marginal value of waiting.

A Rolling Budget (of 1 project)

Fixing duration τ:

(P) Delegate until a project is adopted.

After each project, freeze for duration τ, then start over.

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$$\begin{aligned} \text{Equilibrium} \quad &\iff \quad \text{Agent is willing to resist bad projects} \\ &\iff \quad \tau \geq \bar{\tau} \end{aligned}$$

... Lower bound on $\mathbb{E}(1 - \delta^{\tau})$

Setting $\tau = \bar{\tau}$ is best, yielding

agent value $= \omega$, the *marginal value of waiting*.

Aligned Delegation

In an equilibrium with no bad projects, interests are aligned on-path: *aligned equilibrium*.

Proposition: The $\bar{\tau}$-freeze equilibrium yields highest payoffs among all aligned equilibria.

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Efficiency?

$$\frac{\text{principal's best aligned equilibrium profit}}{\text{principal's first-best profit}} = \frac{\bar{\theta} - \underline{\theta}}{\bar{\theta}} < 1$$

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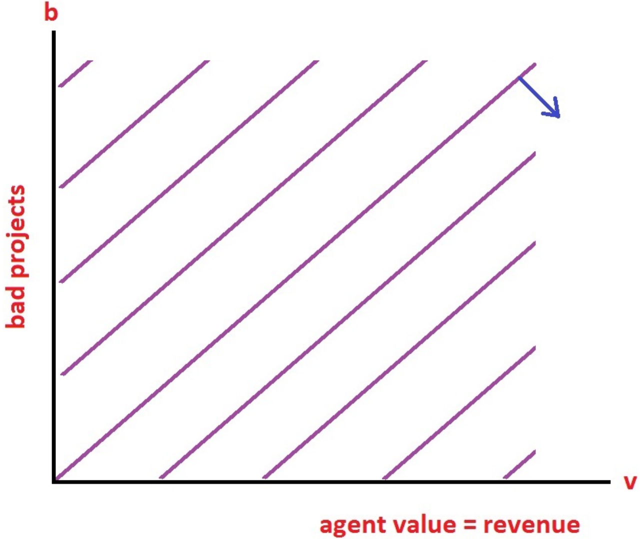
Value Space



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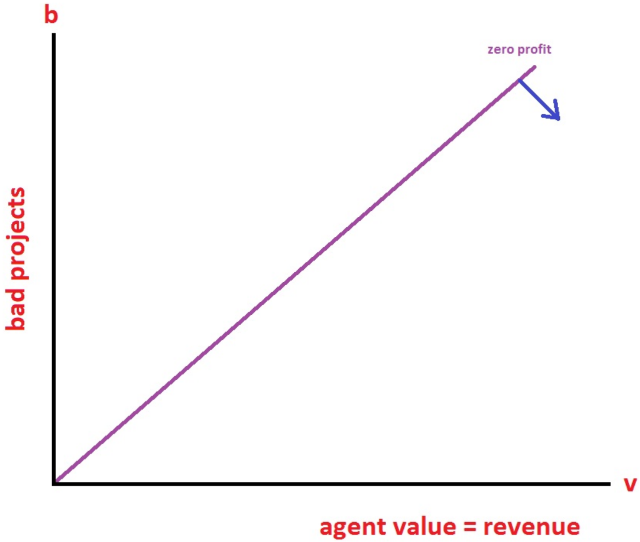

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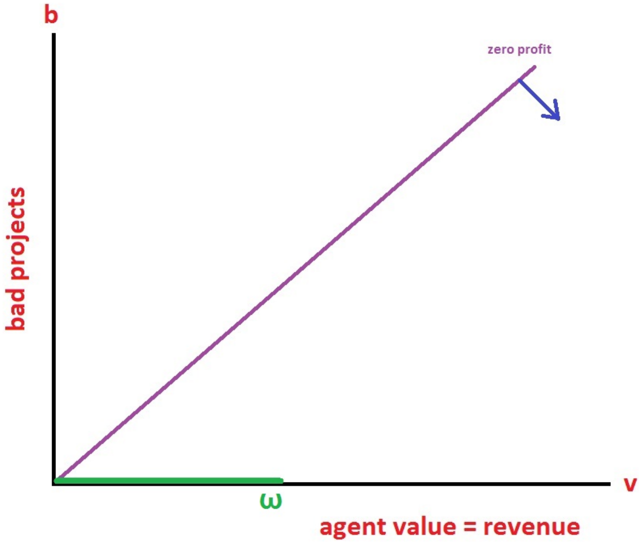
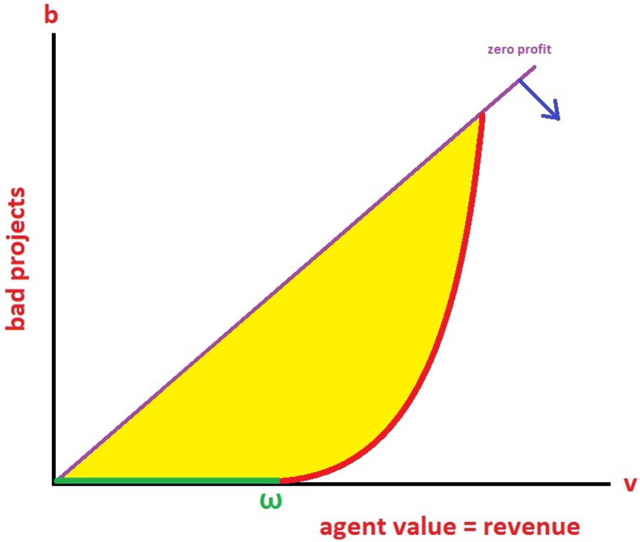


The Equilibrium Value Set



Agent Values: Evolution

Given agent continuation value v:

If today's play is...	***Then tomorrow's value is...***
no projects adopted	$v^F := \dfrac{v}{\delta}$
all projects adopted	$v^P := \dfrac{v - (1-\delta)\mathbb{E}[\theta]}{\delta}$

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all projects adopted	$v^P := \frac{v - (1-\delta)\mathbb{E}[\theta]}{\delta}$
only good projects adopted (agent incentives binding)	v^P if project, $v^{NP} := v^P + \frac{1-\delta}{\delta}\underline{\theta}$ otherwise

Main Theorem: The Equilibrium Value Set

Part 1

There is an agent value $\bar{v}$ and a function $B : [0, \bar{v}] \to \mathbb{R}_+$ such that:

- (v, b) is an equilibrium value pair $\iff v \in [0, \bar{v}]$, $\pi(v, b) \geq 0$, and $b \geq B(v)$.
- B is the lower convex envelope of

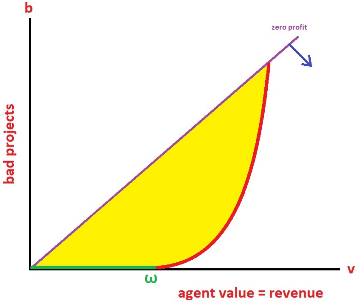
$$v \mapsto \min \left\{ \delta B(v^F),\ (1-h)(1-\delta)+\delta B(v^P),\ \delta \left[hB(v^P) + (1-h)B(v^{NP}) \right] \right\}.$$

Main Theorem: The Equilibrium Value Set

Part 2

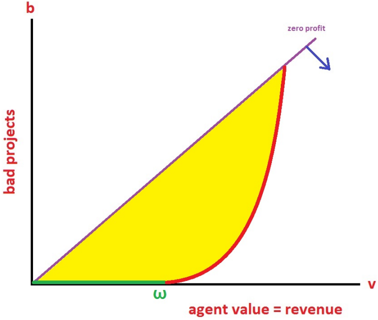
- $B|_{[0,\omega]} = 0$.
- $B(\bar{v}) = (1-\delta)(1-h) + \delta B(\bar{v}^P)$ and $\pi(\bar{v}, B(\bar{v})) = 0$.
- On $[\delta\bar{v} + (1-\delta)\omega,\ \ \bar{v}]$, B is affine.
- On $(\omega,\ \ \delta\bar{v} + (1-\delta)\omega]$, B is strictly convex with

$$B(v) = \delta\left[(1-h)B(v^{NP}) + hB(v^P)\right].$$



The Proof: Some Key Steps

- Principal need never mix.
- $\pi \geq 0$ is sufficient for principal IC.
- Graph(B) is self-generating.



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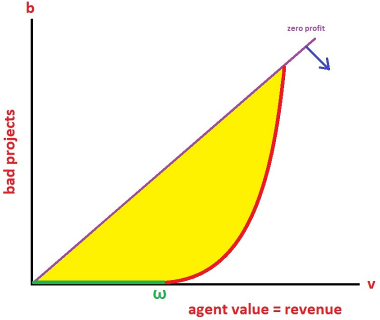

- Agent need never mix.
- Agent need never waste a good project.
- If agent is exercising restraint, IC binds.

The Proof: Some Key Steps, Ctd.

Therefore, B is the convex lower envelope of

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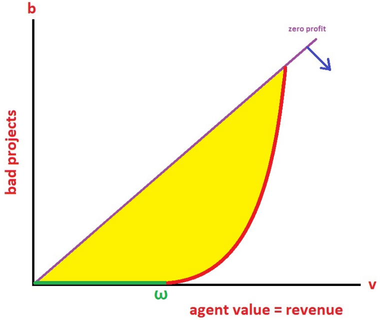
- Freeze is inefficient for values above ω.
- Only admit bad projects at high values, mixing in between.
- Bad project and mixing regions "as small as possible".



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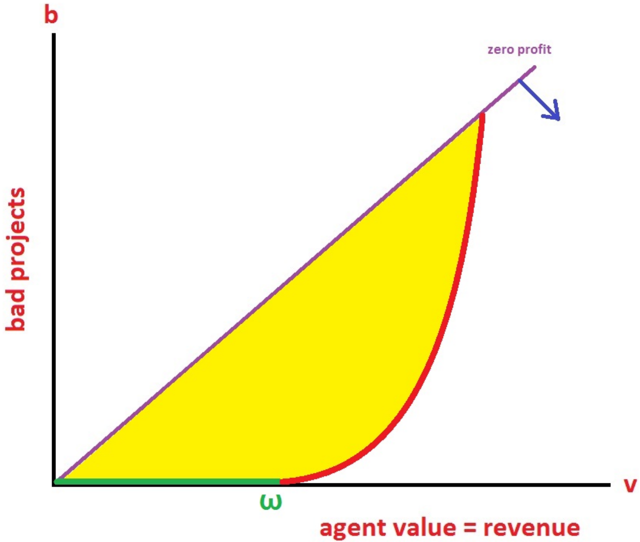

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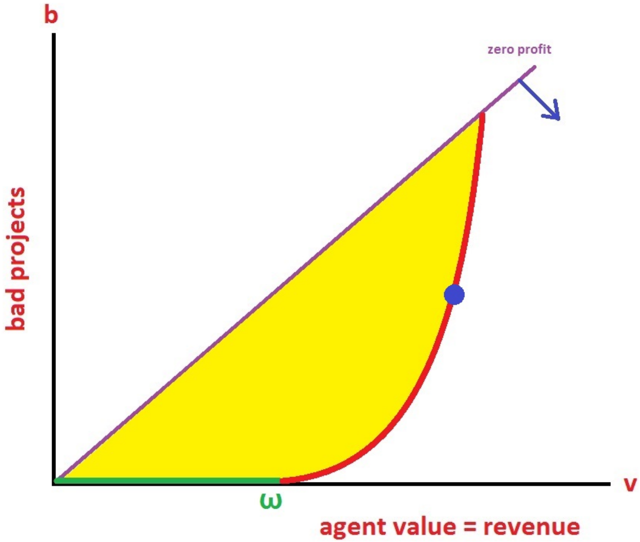
Drifting Toward Conservatism

Theorem:
In any Pareto efficient equilibrium, there is (w.p. 1) some finite time k such that:

- Up to k, every good project taken, but some bad ones too.
- After k, no bad projects taken, but some good ones missed.

Relationship progresses from overfunding to underfunding.

DRIFTING TOWARD CONSERVATISM: PROOF



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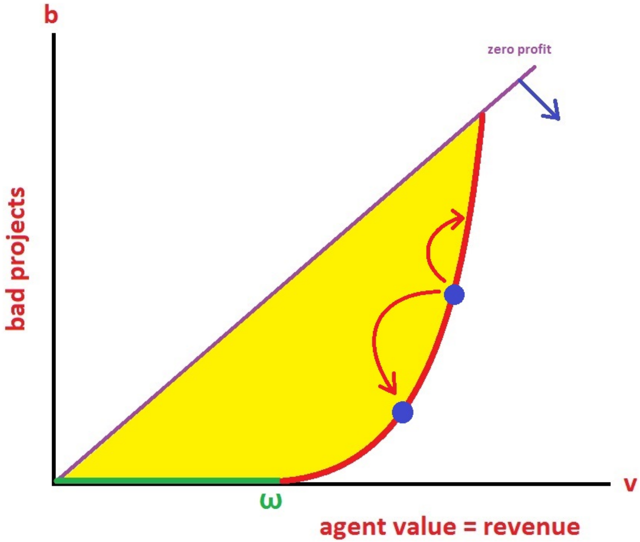

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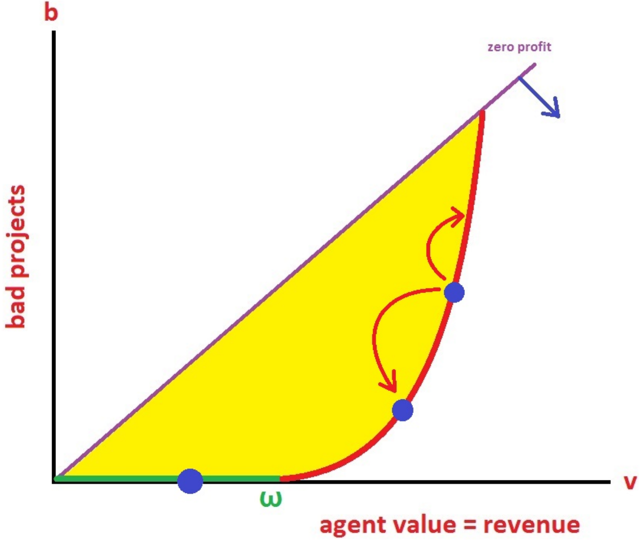

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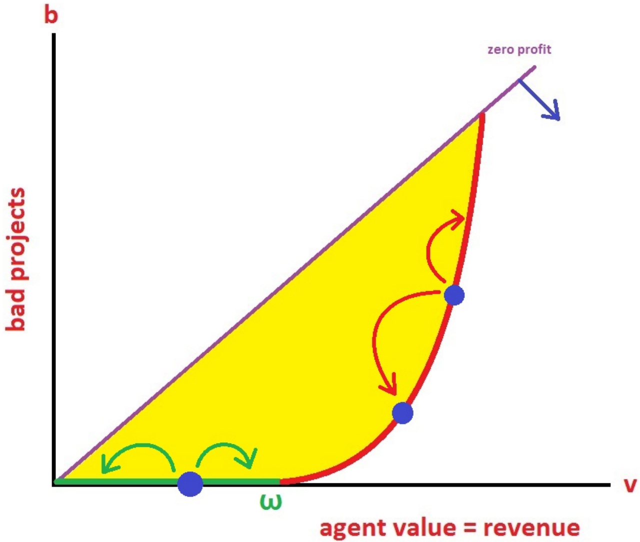

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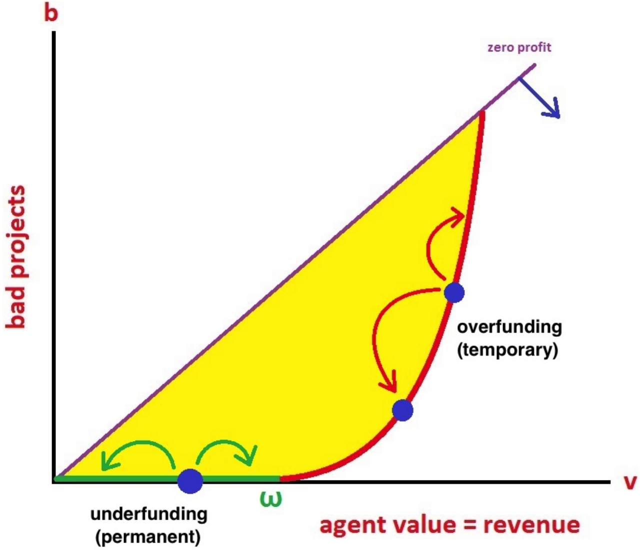

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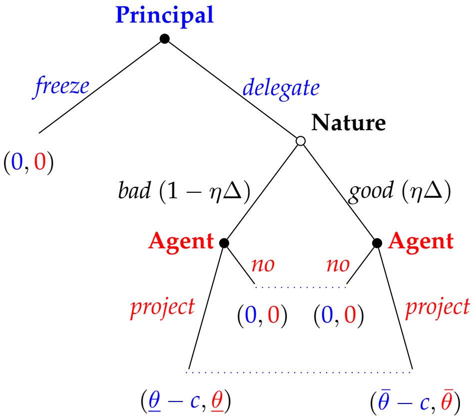
A Continuous-Time Limit

$\delta = 1 - r\Delta$, $h = \eta\Delta$, $\Delta \approx 0$, nonzero payoffs are lump-sums.



Good project Poisson (η), bad projects *always* available.

Dynamics of Pareto Efficient Equilibrium

Continuous-time analogues...

- Marginal value of waiting: $\omega = \eta(\bar{\theta} - \underline{\theta})$.
- $v \leq \omega \implies B(v) = 0$.
- $v = \bar{v} \implies$ immediate bad project, $v^P = v - r\underline{\theta}$.
- $v \in (\omega, \bar{v}) \implies$ only good, $v^P = v - r\underline{\theta}$, $\dot{v}^{NP} = r(v - \omega)$.

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Renormalize...

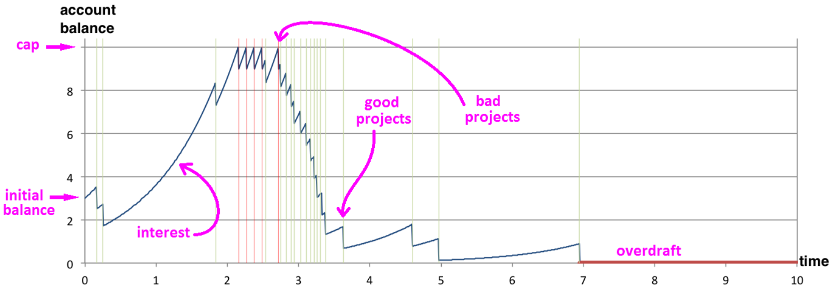
$$x := \frac{v - \omega}{r\underline{\theta}} \textbf{ (balance)}, \qquad \bar{x} := \frac{\bar{v} - \omega}{r\underline{\theta}} \textbf{ (cap)}.$$

Law of motion while $x \in [0, \bar{x})$:

$$x^P = x - 1 \textbf{ (spending budget)}, \quad \dot{x}^{NP} = rx \textbf{ (interest)}.$$

Our "Dynamic Capital Budgeting" Contract

Account Balance Over Time



Theorem: DCB is Optimal

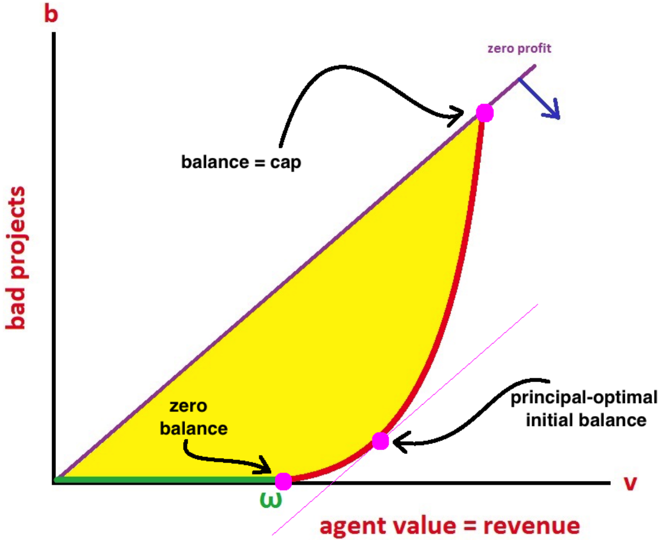
1. There is a largest cap $\bar{x}$ such that a DCB contract with cap $\bar{x}$ is an equilibrium.

2. There is a unique initial balance $x^* > 0$ which maximizes principal profit (given cap $\bar{x}$).

3. All Pareto efficient equilibrium payoffs come from a DCB contract with cap $\bar{x}$ and initial balance in $x \in [x^*, \bar{x}]$.

Our "Dynamic Capital Budgeting" Contract

In Value Space



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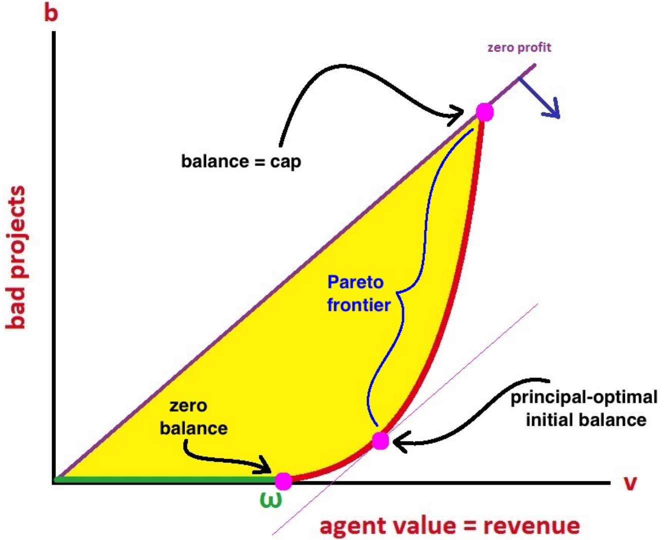


Questions & Answers

What we've seen

1. How productive can the relationship be?

   Full characterization of equilibrium payoff set (at fixed δ).

   If credible, bad projects are always efficiency enhancing.

2. How does the relationship evolve over time?

   Frontloaded reward: first overfund, then underfund.

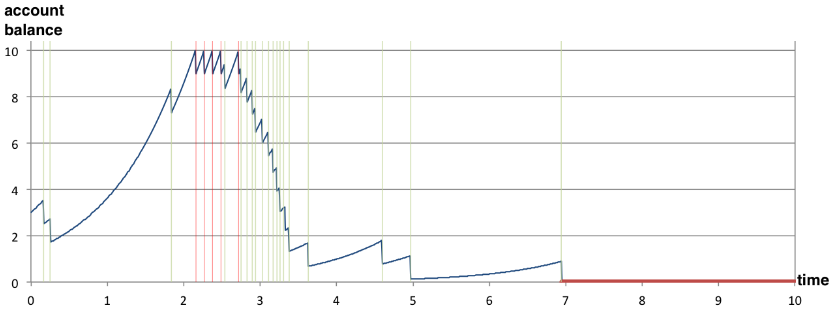
   Commitment drastically changes the long-run dynamics.

3. How should delegation be implemented?

   Budgeting: expense account, market interest rate, cap.

   Simple capital structure as *unique* implementation.

Thanks!



Extensions

Money:

What if the principal can use incentive pay?

Hands-off management:

What if the principal can only *permanently* fire the agent, and must otherwise fully delegate?

Fresh talent:

What if the principal can fire and replace the agent?

Monitoring:

What if the principal sees a very noisy signal ex-post?

The Equilibrium Value Set