Rank Uncertainty in Organizations

Marina Halac
Yale

Elliot Lipnowski
Columbia

Daniel Rappoport
Chicago

Seminars in Economic Theory, September 2020

Motivation

Projects in firms rely on workers' complementary efforts

Workers rewarded for overall success face strategic risk

- Reluctant to work unless they expect others will

What is firm's optimal scheme that uniquely implements work?

1. Is transparency about workers' rank and pay good?
2. Is pay inequality a feature of optimal incentives?

Ensuring effort

Want to provide assurance to worker that coworker will work

Winter (2004): Specify hierarchy of workers

- High reward for high-rank workers to always work
- Lower for low-rank workers to work when higher-rank do

Scheme is discriminatory

But this assumes public contracts

- Realistic? Optimal?

Rank uncertainty

Current debate: lack of transparency in firms

- Firms rarely (internally) disclose employee contract terms
- Also discourage/prohibit workers from discussing terms
- Secrecy further sustained by social norms

We show firm's optimal scheme indeed limits information

- Create rank uncertainty to address strategic uncertainty

Scheme is unique and entails no discrimination

Literature

Some highlights

Contracting with externalities

- Segal (1999, 2003), Winter (2004)
- Randomization: Eliaz-Spiegler (2015), Moriya-Yamashita (2019)

Information design

- Inostroza-Pavan (2020), Hoshino (2019), Mathevet-Perego-Taneva (2020), Morris-Oyama-Takahashi (2020)

Broader literature on incentives and discrimination

Model

Moral hazard in teams

Timeline:

- Principal offers contracts to set of agents (see next slide)
- Each agent works or shirks
- Project succeeds or fails
- Based on outcome, agents paid according to contracts

Want everybody to work, at low monetary cost

Parameters:

$$
\begin{aligned}
\text{Agents} &: \quad N = \{1, \ldots, N\} \\
\text{Production} &: \quad P : 2^N \to [0,1] \text{ supermodular and increasing} \\
\text{Costs} &: \quad \vec{c} = (c_i)_i, \text{ all } > 0
\end{aligned}
$$

Optimal contracting problem

Before play, principal designs **incentive scheme** $\sigma = \langle T, q, B \rangle$:

- $T = \prod_i T_i$, where each T_i is finite
- $q \in \Delta T$
- $B = (B_i)_i$, where $B_i : T_i \to \mathbb{R}_+$ is i's bonus from success

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Say σ **uniquely implements work** (UIW) if $\forall \epsilon > 0$, everybody works in every BNE of the Bayesian game with type space $\langle T, q \rangle$ and bonuses $B + \epsilon$.

Principal's problem:

$$\begin{aligned} W_* = \quad & \inf_\sigma \quad \text{Expected total payment} \\ & \text{s.t.} \quad \sigma \text{ UIW} \end{aligned}$$

A Simple Example

Winter (2004), leading example

Review of "public contracts" case

$$2 \text{ agents, } c_i = c, \text{ project succeeds w.p. } \begin{cases} 1 & : \text{both work} \\ p^2 & : \text{both shirk} \\ p & : \text{one each} \end{cases}$$

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To make all-work an equilibrium, pay each worker

$$b_L := \frac{c}{1-p}$$

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$$b_L := \frac{c}{1-p}$$

To make it the *only* equilibrium, pay one worker

$$b_H := \frac{c}{p(1-p)} > b_L$$

... but can then pay other worker b_L

Winter (2004), leading example

Limiting transparency

2 agents, $c_i = c$, project succeeds w.p. $\begin{cases} 1 & \text{: both work} \\ p^2 & \text{: both shirk} \\ p & \text{: one each} \end{cases}$

Suppose we privately offer one worker a random contract:

$$b_H \text{ or } b_L, \text{ each w.p. } \frac{1}{2}$$

Offer the other worker $b_M := \dfrac{c}{\frac{1}{2}p(1-p) + \frac{1}{2}(1-p)}$

WINTER (2004), LEADING EXAMPLE

LIMITING TRANSPARENCY

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Agents "reassure" each other $\implies$ both work

Winter (2004), leading example

Effects of limiting transparency

$b_M < \frac{1}{2} b_H + \frac{1}{2} b_L$

$\implies$ Total average payments decrease

$\implies$ Public contracts *with* loss of generality

$b_L < b_M < b_H$

$\implies$ Less transparency can mean less discrimination

Ranking Agents

Ranking schemes

$\sigma = \langle T, q, B \rangle$ is a **ranking scheme** if:

1. Every distinct i, j have $q\{t : t_i = t_j\} = 0$;
2. Every i and t_i have

$$B_i(t_i)\, \mathbb{E}_q\left[P\{j : t_j \le t_i\} - P\{j : t_j < t_i\} \,\middle|\, t_i \right] = c_i.$$

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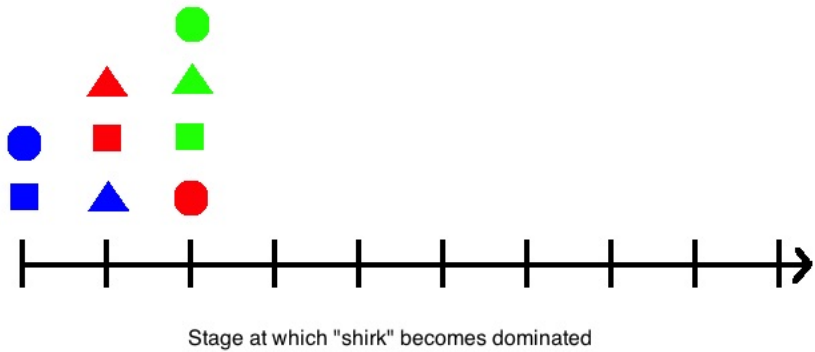
Lemma:

1. Every ranking scheme UIW.
2. Anything that UIW is costlier than some ranking scheme.

So principal can optimize over ranking schemes

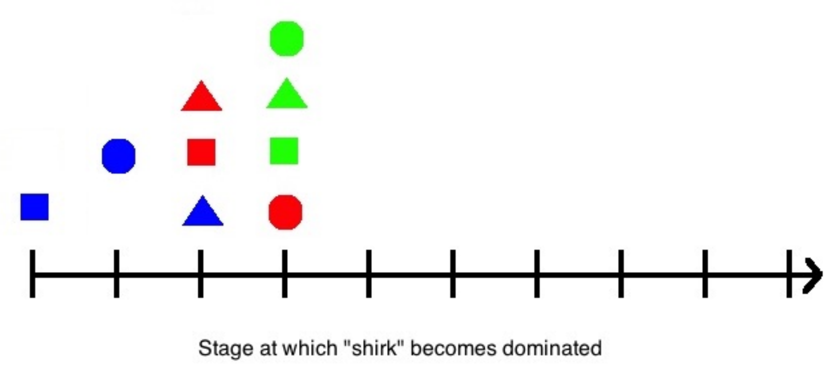
RANKING SCHEMES

CONSTRUCTIVE PROOF



Ranking schemes

Constructive proof



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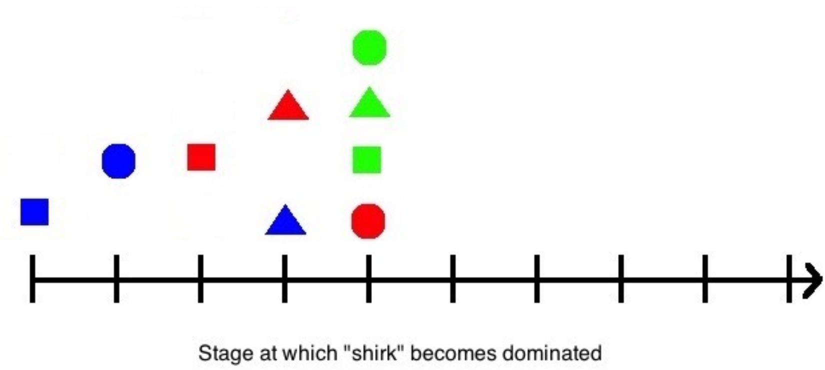

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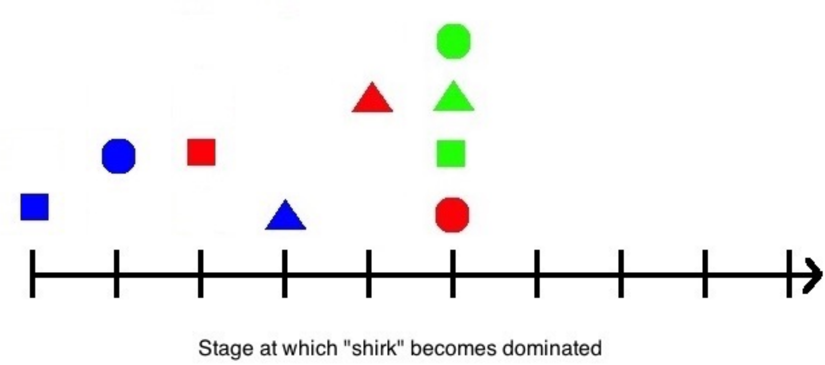

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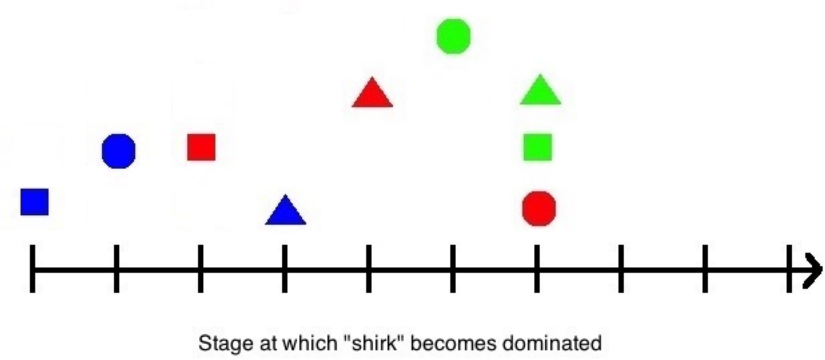

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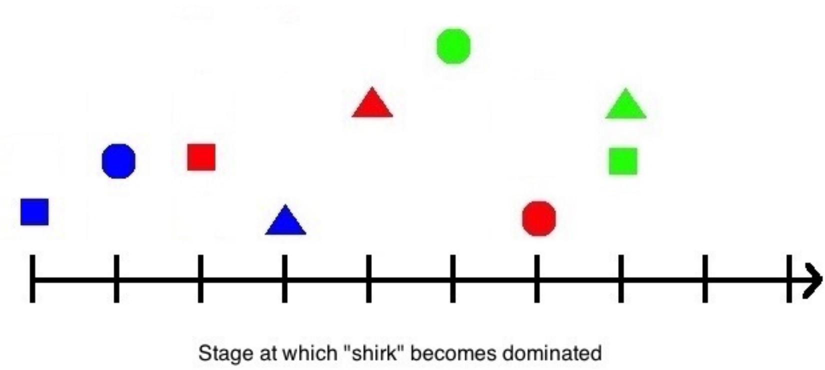

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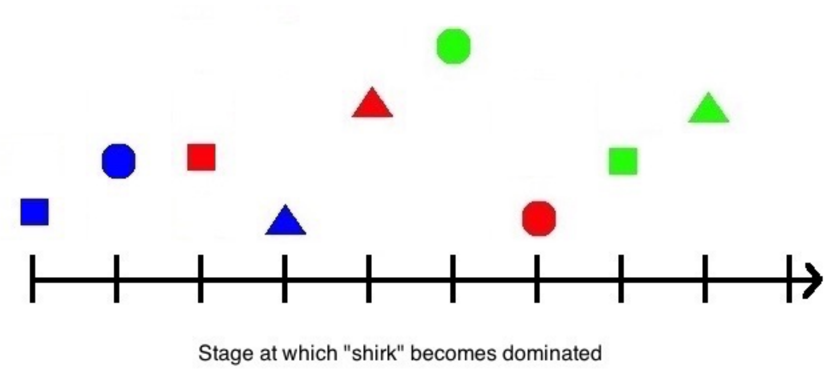

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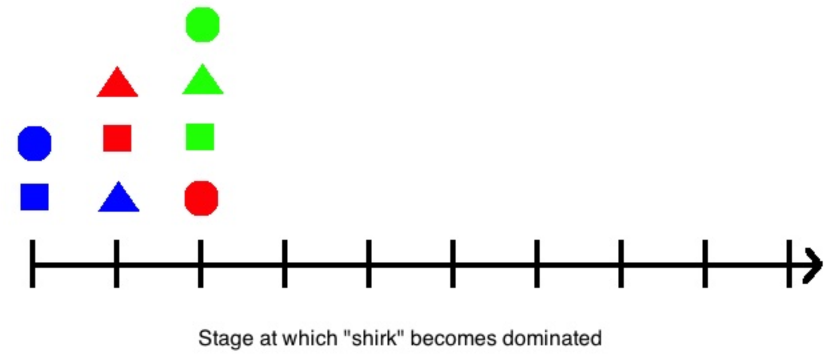

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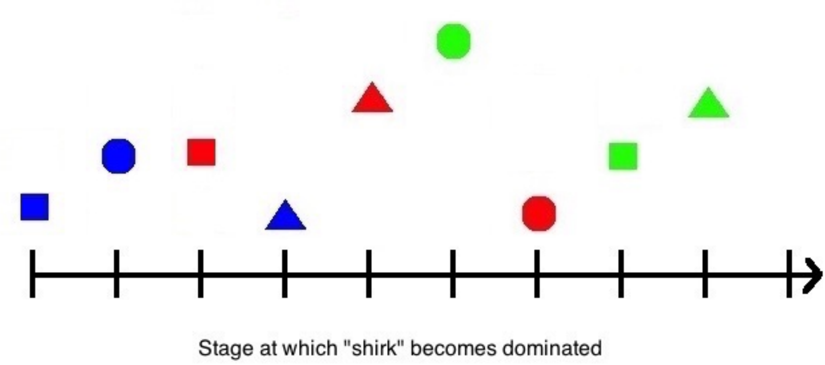

Solving the Principal's Problem

Incentive costs of a ranking

Let Π be the set of permutations on N.

- Each t (without ties) induces an **agent ranking** $\pi(t) \in \Pi$
- Ranking scheme σ induces **ranking distribution** $\mu^{\sigma} \in \Delta\Pi$
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Given $\mu_i \in \Delta\Pi$, let

$$\frac{c_i}{\mathbb{E}_{\pi \sim \mu_i}\left[P\{j : \ \pi_j \leq \pi_i\} - P\{j : \ \pi_j < \pi_i\}\right]}$$

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(Very Easy) Lemma: A ranking scheme $\sigma = \langle T, q, B \rangle$ costs

$$\sum_i \mathbb{E}_{t_i\sim q_i} f_i\left(\mu_i^{\sigma}(\cdot|t_i)\right)$$

The optimal value

$$f_i(\mu_i) := \frac{c_i P(N)}{\mathbb{E}_{\pi \sim \mu_i}\left[P\{j\colon \pi_j \leq i\} - P\{j\colon \pi_j < i\}\right]}$$

Theorem 1: The firm's optimal value is:

$$\min_{\mu \in \Delta\Pi} \sum_i f_i(\mu).$$

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Theorem 1: The firm's optimal value is:

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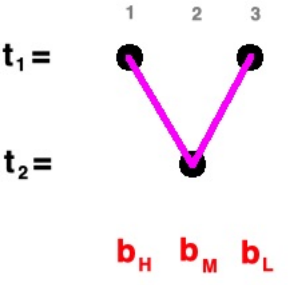
Proof: First, $\mathbb{E} f_i\left(\mu_i^\sigma(\cdot|t_i)\right) \geq f_i\left(\mathbb{E}\mu_i^\sigma(\cdot|t_i)\right) = f_i(\mu^\sigma)$

$\implies$ can't do better

The optimal value

Constructive proof



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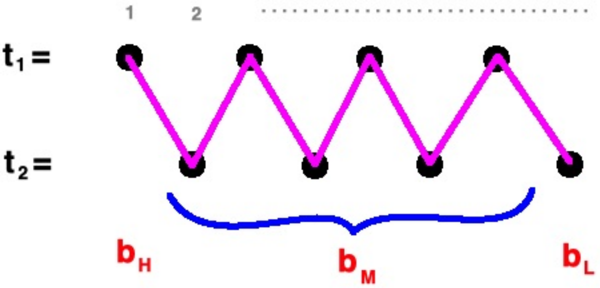

THE OPTIMAL VALUE

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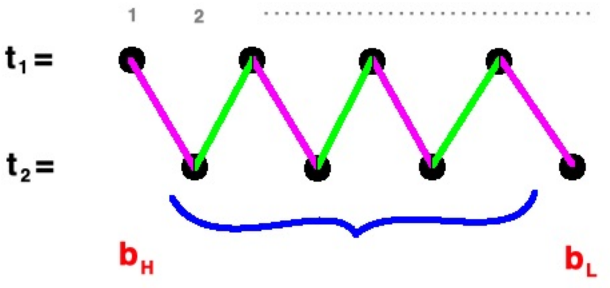

Optimal incentive schemes

Auxiliary program characterizes optimal incentive schemes, and generates uniquely optimal payments

Theorem 2:

1. There is a unique bonus profile $b^* \in \mathbb{R}^N$ which minimizes $\sum_{i \in N} b_i$ among all

$$b \in \left\{ \frac{1}{P(N)} \left(f_1(\mu), \ldots, f_N(\mu) \right) : \ \mu \in \Delta\Pi \right\}.$$

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Proof idea: f ~ strictly convex, and $\exists$ common prior over Π

Discrimination

From Theorem 2, **optimal bonus to i** uniquely pinned down.

Corollary:

If $c_i = c_j$ and $P(J \cup \{i\}) = P(J \cup \{j\})\ \forall J \subseteq N \setminus \{i, j\}$, then $b_i^* = b_j^*$.

Every optimal $(\sigma^m)_m$ has $\mathbb{P}^m\{|b_i - b_j| < \epsilon\} \to 1\ \forall \epsilon > 0$.

- No discrimination between identical agents

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- No discrimination between identical agents
- Little discrimination between similar agents
- Rank uncertainty *strictly* optimal for similar agents

Heterogeneity and Rank Uncertainty

Emergence of a hierarchy

Proposition: Suppose $P(J) = P(|J|)$, and label $c_1 \leq \cdots \leq c_N$.

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- Have $1 \succ \cdots \succ N$ iff

$$\frac{P(1) - P(0)}{\sqrt{c_1}} \geq \cdots \geq \frac{P(N) - P(N-1)}{\sqrt{c_N}}$$

  and $1 \sim \cdots \sim N$ iff

$$\frac{P(1) - P(0)}{\sum_{i=1}^{1} \sqrt{c_i}}, \ldots, \frac{P(N-1) - P(0)}{\sum_{i=1}^{N-1} \sqrt{c_i}} < \frac{P(N) - P(0)}{\sum_{i=1}^{N} \sqrt{c_i}}$$

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- Payment to i increases in c_i or c_j.
  Markup for i decreases in c_i.

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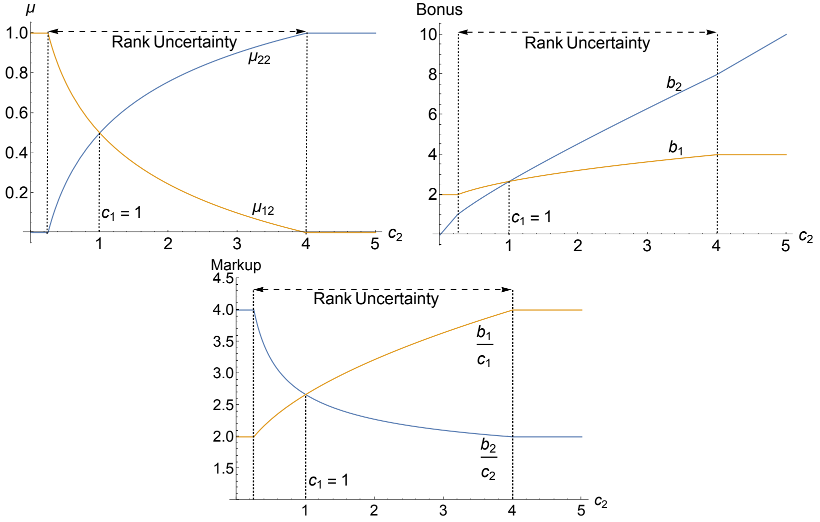

Extensions

Substitutable effort

Workers' efforts may not always be complements

- Congestion costs; small tasks

Modify analysis for uniquely (IC)Rationalizable work:

$$f_i(\mu_i) = \frac{c_i P(N)}{\mathbb{E}_{\pi \sim \mu_i} m_i\left(\{j \in N : \ \pi_j < \pi_i\}\right)}$$

where $m_i(J) := \min\left\{P\left(\hat{J} \cup \{i\}\right) - P(\hat{J}) : \ J \subseteq \hat{J} \subseteq N \setminus \{i\}\right\}$

Equilibrium analysis seems harder

INFORMATION SHARING

Rank uncertainty undermined if agents can *reveal terms*

Revealing equilibrium with verifiable disclosure (symmetric P)

- "Threat beliefs" punish nondisclosure
- Rank uncertainty cannot be ensured!
- Such a firm cannot outperform Winter (2004)

Firms discourage/prohibit discussion about contracts

- Gely-Bierman (2003), Hegewisch-Williams-Drago (2011)
- Edwards (2005), Cullen-Perez-Truglia (2018)

Interdependent contracting

So far: workers know own contractual terms

What if (effective) contractual terms are interrelated?

- In math, $B_i(t)$ rather than $B_i(t_i)$
- Could arise through discretionary pay

Punchline: Strategic risk becomes irrelevant

- Minimum bonus b_i to make work an equilibrium
- $t_i \in \{1, 2\}$ i.i.d. where type 1 has probability $\epsilon \approx 0$

$$B_i(t) := \begin{cases} \frac{c_i}{P\{i\} - P(\varnothing)} & : \ t_i = 1, \\ \frac{1}{\epsilon^{N-1}} \cdot b_i & : \ t_i = 2 \text{ and } t_j = 1 \text{ for all } j \neq i, \\ 0 & : \ \text{otherwise} \end{cases}$$

Concluding remarks

Rank uncertainty allows firm to ensure work at lower cost

Current debate: increase transparency to reduce discrimination

- Regulation protecting workers who share contract terms
- Some firms moving to open (internal) disclosure

We find: discrimination optimal $\leftrightsquigarrow$ public contracts

Either measures will be counterproductive, or factors other than optimal incentives are behind firms' discrimination

Thanks!