Peer-Confirming Equilibrium

Elliot Lipnowski
University of Chicago

Evan Sadler
Columbia University

T@PS, December 2018

Strategic uncertainty in games

Optimal actions often depend on what others do

- Information about others' strategies is important
- Players may face strategic uncertainty

Strategic uncertainty in games

Optimal actions often depend on what others do

- Information about others' strategies is important
- Players may face strategic uncertainty

Standard solution concepts make extreme assumptions

- Nash equilibrium (no uncertainty)
- Rationalizability (total uncertainty)

Strategic uncertainty in games

Optimal actions often depend on what others do

- Information about others' strategies is important
- Players may face strategic uncertainty

Standard solution concepts make extreme assumptions

- Nash equilibrium (no uncertainty)
- Rationalizability (total uncertainty)

What play make sense when players have partial information?

Social networks and strategic knowledge

Our approach: social network encodes epistemic structure

Routine interactions with friends shape our expectations

More accurate conjectures about friends/neighbors/coworkers than about strangers

PCE explicitly models strategic knowledge via social ties

Peer-confirming equilibrium

Augment game with a social network

Peer-confirming equilibrium

Augment game with a social network

Profile is a PCE if

- Players respond optimally to conjectures on others' play
- Players' conjectures about neighbors are correct
- Above facts are common belief

Peer-confirming equilibrium

Augment game with a social network

Profile is a PCE if

- Players respond optimally to conjectures on others' play
- Players' conjectures about neighbors are correct
- Above facts are common belief

Complete network $\implies$ Nash equilibrium

Empty network $\implies$ rationalizability

More links $\implies$ more refined prediction

Roadmap

PCE in simultaneous-move games

Examples

- Role of central players
- Protests and elite coordination

PCE in dynamic games

- Actions can signal *others'* plans
- Can get refinement of both SPE and EFR

Related Work

Epistemic game theory

- e.g. Battigalli and Siniscalchi (2002)

Protest games

- e.g. Angeletos et al. (2007)

Related Work

Epistemic game theory

- e.g. Battigalli and Siniscalchi (2002)

Protest games

- e.g. Angeletos et al. (2007)

Related solution concepts

- RCE (Rubinstein and Wolinsky, 1994)
- RPCE (Fudenberg and Kamada, 2015)

PCE in simultaneous-move games

Simultaneous-move game of complete information:

- Set of players N (finite)
- Strategies S_i for player i, $S = \prod_{i \in N} S_i$ (measurable)
- Payoff u_i for player i (bounded, measurable)

PCE in simultaneous-move games

Simultaneous-move game of complete information:

- Set of players N (finite)
- Strategies S_i for player i, $S = \prod_{i \in N} S_i$ (measurable)
- Payoff u_i for player i (bounded, measurable)

Augment with undirected graph (N, G)

- Write G_i for neighbors of i

Conjectures and best replies

Strategy $s_i^* \in S_i$ is a best reply to conjecture $\mu_i \in \Delta(S_{-i})$ if

$$s_i^* \in \arg\max_{s_i \in S_i} \int_{S_{-i}} u_i(s_i, \cdot)\, \mathrm{d}\mu_i$$

Set of best replies $r_i(\mu_i)$

Conjectures and best replies

Strategy $s_i^* \in S_i$ is a best reply to conjecture $\mu_i \in \Delta(S_{-i})$ if

$$s_i^* \in \arg\max_{s_i \in S_i} \int_{S_{-i}} u_i(s_i, \cdot)\, \mathrm{d}\mu_i$$

Set of best replies $r_i(\mu_i)$

Given $\sigma \in S$, define

$$S_{-i}^{\sigma,G} = \{s_{-i} \in S_{-i} \,:\, s_j = \sigma_j,\ \forall\, j \in G_i\},$$

profiles consistent with i's knowledge at σ

Conjectures and best replies

Given $\sigma \in \Sigma \subseteq S$, define

$$\Delta_i^{\sigma,G}(\Sigma) = \left\{\mu_i \in \Delta(S_{-i}) \ : \ \mu_i(\Sigma_{-i}) = \mu_i\left(S_{-i}^{\sigma,G}\right) = 1\right\}$$

viable conjectures relative to Σ at σ

Definition of PCE: simultaneous moves

Network-consistent best replies to Σ

$$B_G(\Sigma) = \{\sigma \in \Sigma \;:\; \forall i \in N,\; \exists\, \mu_i \in \Delta_i^{\sigma,G}(\Sigma) \text{ s.t. } \sigma_i \in r_i(\mu_i)\}$$

Definition of PCE: simultaneous moves

Network-consistent best replies to Σ

$$B_G(\Sigma) = \{\sigma \in \Sigma \,:\, \forall i \in N,\ \exists\, \mu_i \in \Delta_i^{\sigma,G}(\Sigma) \text{ s.t. } \sigma_i \in r_i(\mu_i)\}$$

Definition:
A profile σ is a **peer-confirming equilibrium** if there exists $\Sigma \subseteq S$ such that $\sigma \in \Sigma \subseteq B_G(\Sigma)$.

Definition of PCE: simultaneous moves

Network-consistent best replies to Σ

$$B_G(\Sigma) = \{\sigma \in \Sigma \,:\, \forall i \in N,\ \exists\, \mu_i \in \Delta_i^{\sigma,G}(\Sigma) \text{ s.t. } \sigma_i \in r_i(\mu_i)\}$$

Definition:
A profile σ is a **peer-confirming equilibrium** if there exists $\Sigma \subseteq S$ such that $\sigma \in \Sigma \subseteq B_G(\Sigma)$.

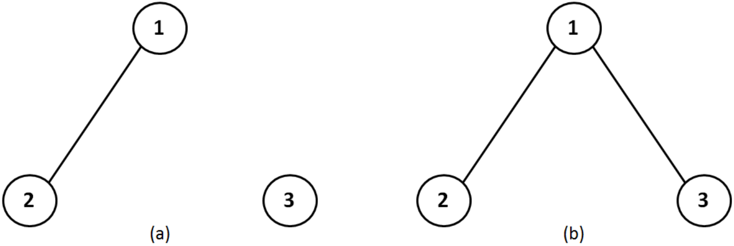
If each S_i compact and u_i continuous, equivalent definition is

$$PCE \equiv \bigcap_{k=0}^{\infty} B_G^k(S)$$

Toy example 1

An investment game

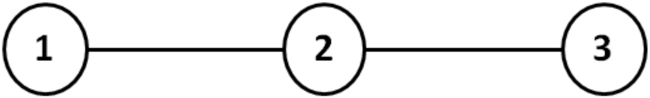
- Each of 3 players can invest at cost $c \in \left(\frac{1}{2}, 1\right)$
- If at least one invests, generate unit of surplus
- Divide surplus evenly between investors



Toy example 2

Follow the leader

- Each player can choose action 0 or 1
- Player 1 is indifferent between the two
- Others earn payoff 1 iff they match player 1



Payoff relevance

Say i is payoff-relevant to j if there is some $s_{-i} \in S_{-i}$ such that $u_j(\cdot, s_{-i}) : S_j \to \mathbb{R}$ is not constant

Payoff relevance

Say i is payoff-relevant to j if there is some $s_{-i} \in S_{-i}$ such that $u_j(\cdot, s_{-i}) : S_j \to \mathbb{R}$ is not constant

Proposition:
Let $\tilde{G}$ be the payoff relevance network.

- If $G \cap \tilde{G} = \tilde{G}$, then PCE = Nash.
- If $G \cap \tilde{G} = \varnothing$, then PCE = Rationalizability.

Payoff relevance

Say i is payoff-relevant to j if there is some $s_{-i} \in S_{-i}$ such that $u_j(\cdot, s_{-i}) : S_j \to \mathbb{R}$ is not constant

Proposition:
Let $\tilde{G}$ be the payoff relevance network.

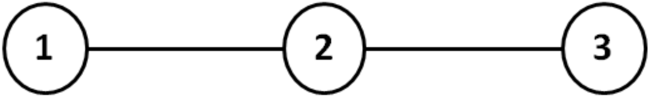
- If $G \cap \tilde{G} = \tilde{G}$, then PCE = Nash.
- If $G \cap \tilde{G} = \varnothing$, then PCE = Rationalizability.

Is $G \cap \tilde{G}$ all that matters?

Toy example 2

Follow the leader

- Each player can choose action 0 or 1
- Player 1 is indifferent between the two
- Others earn payoff 1 iff they match player 1



Consider removing the link between 2 and 3

Public good provision

Population size N, player i invests $x_i \in [0, 1]$ in a public good

Payoffs

$$u_i(x) = 2\sqrt{\sum_{j \in N} x_j} - x_i$$

Public good provision

Population size N, player i invests $x_i \in [0, 1]$ in a public good

Payoffs

$$u_i(x) = 2\sqrt{\sum_{j \in N} x_j} - x_i$$

NE iff total investment is 1; any profile rationalizable

Public good provision

Population size N, player i invests $x_i \in [0, 1]$ in a public good

Payoffs

$$u_i(x) = 2\sqrt{\sum_{j \in N} x_j} - x_i$$

NE iff total investment is 1; any profile rationalizable

Subset M is independent if no two players in M adjacent

Player i is fully connected if all other players link to i in G

Public good provision

Proposition:

Lowest total investment is:

1 if some player is fully connected, 0 if none is.

Highest total investment is:

$|M|$, where M is a largest independent set.

Public good provision

Proposition:

Lowest total investment is:

1 if some player is fully connected, 0 if none is.

Highest total investment is:

$|M|$, where M is a largest independent set.

Can have significant over-investment

- Sparser network allows more investment

Public good provision

Proposition:

Lowest total investment is:

1 if some player is fully connected, 0 if none is.

Highest total investment is:

$|M|$, where M is a largest independent set.

Can have significant over-investment

- Sparser network allows more investment

If a fully connected player invests, get total investment 1

- Fully connected player signals optimal play
- End up in a Nash equilibrium

A protest game

N players simultaneously choose whether to protest or not

Non-protesters earn 0

If at least M protest, leadership is overthrown, protesters ☺

If fewer than M protest, suffer repression, protesters ☹

Assume $2 < M < N$

A protest game

Proposition:

If there exists a fully connected player, all players choose the same action in any PCE.

If $G_i \cup G_j = N$, then i and j choose the same action in any PCE.

A protest game

Proposition:

If there exists a fully connected player, all players choose the same action in any PCE.

If $G_i \cup G_j = N$, then i and j choose the same action in any PCE.

Proof: WLOG, nobody else neighbors with both i and j

A protest game

Proposition:

If there exists a fully connected player, all players choose the same action in any PCE.

If $G_i \cup G_j = N$, then i and j choose the same action in any PCE.

Proof: WLOG, nobody else neighbors with both i and j

Assume i protests and j doesn't

A protest game

Proposition:

If there exists a fully connected player, all players choose the same action in any PCE.

If $G_i \cup G_j = N$, then i and j choose the same action in any PCE.

Proof: WLOG, nobody else neighbors with both i and j

Assume i protests and j doesn't

$\underline{m}_i$ minimum number of i's neighbors that protest in such a PCE

$\overline{m}_j$ maximum number of j's neighbors that protest in such a PCE

A PROTEST GAME

Proposition:

If there exists a fully connected player, all players choose the same action in any PCE.

If $G_i \cup G_j = N$, then i and j choose the same action in any PCE.

Proof: WLOG, nobody else neighbors with both i and j

Assume i protests and j doesn't

$\underline{m}_i$ minimum number of i's neighbors that protest in such a PCE

$\overline{m}_j$ maximum number of j's neighbors that protest in such a PCE

i's incentives in $\underline{m}_i$ PCE $\implies \underline{m}_i + \overline{m}_j \geq M - 1$

j's incentives in $\overline{m}_j$ PCE $\implies \underline{m}_i + \overline{m}_j < M - 1$

PCE in dynamic games

Same idea, more details

Define for multistage games of observable action

PCE in dynamic games

Same idea, more details

Define for multistage games of observable action

Players form conjectures on others' strategies

- Conjectures are history-dependent and Bayesian
- Play sequential best reply to conjectures
- Conjectures on neighbors' future play are correct
- Common strong belief in the above (forward induction)

PCE in dynamic games

Same idea, more details

Define for multistage games of observable action

Players form conjectures on others' strategies

- Conjectures are history-dependent and Bayesian
- Play sequential best reply to conjectures
- Conjectures on neighbors' future play are correct
- Common strong belief in the above (forward induction)

Specializes to

- Subgame perfect equilibrium, if G is complete
- Extensive form rationalizability, if G is empty

The protest game, revisted

Two periods:

1. Leader publicly commits to protest or not
2. All others simultaneously decide whether to protest

The protest game, revisted

Two periods:

1. Leader publicly commits to protest or not
2. All others simultaneously decide whether to protest

Any profile is rationalizable

The protest game, revisted

Two periods:

1. Leader publicly commits to protest or not
2. All others simultaneously decide whether to protest

Any profile is rationalizable

There exist SPE with and without successful protests

A new type of forward induction

Consider PCE in a star network centered on leader

- Leader knows the true strategy profile
- If leader commits to protest, others infer it will succeed
- Therefore, leader always protests, all others follow

A new type of forward induction

Consider PCE in a star network centered on leader

- Leader knows the true strategy profile
- If leader commits to protest, others infer it will succeed
- Therefore, leader always protests, all others follow

Choice of one player can signal intentions of *other* players

A new type of forward induction

Consider PCE in a star network centered on leader

- Leader knows the true strategy profile
- If leader commits to protest, others infer it will succeed
- Therefore, leader always protests, all others follow

Choice of one player can signal intentions of *other* players

Joint identifying assumption

- i is rational
- i has correct beliefs about neighbors' play

A new type of forward induction

Consider PCE in a star network centered on leader

- Leader knows the true strategy profile
- If leader commits to protest, others infer it will succeed
- Therefore, leader always protests, all others follow

Choice of one player can signal intentions of *other* players

Joint identifying assumption

- i is rational
- i has correct beliefs about neighbors' play

This type of signaling can refine both EFR and SPE

What we've seen

PCE uses social relationships to refine predictions in games

Network structure has nuanced implications

- Role of central players sensitive to payoff structure
- Signaling of strategic information in dynamic games

Portable, interpretable model for partial strategic uncertainty