Addressing Strategic Uncertainty with Incentives and Information

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SETTING

Principal contracts with set of agents

Can provide information about fundamentals

Can provide information on others' contracts and information

Optimal scheme to uniquely implement desired action profile?

Contribution

Addressing strategic uncertainty

1. Incentive design
   - Segal (2003), Winter (2004), Bernstein-Winter (2012), Chassang-Del Carpio-Kapon (2020), Halac-Kremer-Winter (2020, 2021), Camboni-Porcellacchia (2021), Halac-Lipnowski-Rappoport (2021)
2. Information design
   - Hoshino (2019), Moriya-Yamashita (2019), Mathevet-Perego-Taneva (2020), Morris-Oyama-Takahashi (2020), Inostroza-Pavan (2021), Li-Song-Zhao (2021)

Today: Methodology to jointly study both instruments

Model

Environment

Parameters:

$$\begin{aligned}\text{Agents} &: \quad N = \{1, \ldots, N\} \\ \text{Fundamental states} &: \quad p_0 \in \Delta\Omega \\ \text{Allocations} &: \quad (X_i)_{i\in N} \\ \text{Agent preferences} &: \quad u_i : \{0,1\}^N \times X_i \times \Omega \to \mathbb{R}\end{aligned}$$

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Timeline:

- Principal designs contracts + information (see next slide)
- Then, agents simultaneously choose from $\{0, 1\}$

Want to maximize $\mathbb{E} \sum_i v_i$ subject to everyone choosing 1.

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Assumption: Some $\bar{x}_i \in X_i$ makes 1 dominant for i

Optimal contracting problem

Before play, principal designs **incentive scheme** $\sigma = \langle T, q, \chi \rangle$:

- $T = \prod_i T_i$, where each T_i is finite
- $q \in \Delta(T \times \Omega)$ with $\text{marg}_{\Omega} q = p_0$
- $\chi = (\chi_i)_i$, where $\chi_i : T_i \to X_i$ is i's allocation

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Principal's problem:

$$\begin{aligned} \sup_\sigma \quad & \mathbb{E} \sum_i v_i(\chi_i(t_i), \omega) \\ \text{s.t.} \quad & \sigma \text{ is UIF} \end{aligned}$$

Solving the principal's problem

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Given such t, the **ranking state** is $\pi(t) \in \Pi$ induced by $(t_i^R)_i$

The **total state** is $(\pi(t), \omega) \in \Pi \times \Omega$

Each type has **belief** $\mu_i^q(\cdot|t_i) \in \Delta(\Pi \times \Omega)$

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Ranking-consistent behavior $A_{-i}(\pi) \subseteq A_i$ where

- $\{j : \pi_j < \pi_i\}$ choose 1
- $\{j : \pi_j > \pi_i\}$ could choose anything

$I_i(x_i, \pi, \omega) = \min_{a_{-i} \in A_{-i}(\pi)}[u_i(1, a_{-i}, x_i, \omega) - u_i(0, a_{-i}, x_i, \omega)]$

$\mathcal{X}_i^*(\mu_i) = \{x_i : \mathbb{E} I_i(x_i, \cdot) > 0\}$ allocations uniquely incentivizing 1

Ranking schemes

$\sigma = \langle T, q, B \rangle$ is a **(strict) ranking scheme** if

1. Every supported t has no ties
2. Every i and t_i have $\chi_i(t_i) \in \mathcal{X}_i^* \left(\mu_i^q(\cdot|t_i) \right)$

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Lemma:

1. Every ranking scheme is UIF
2. Any UIF scheme is payoff-equivalent to a ranking scheme

So principal can optimize over ranking schemes

The optimal value

$$
\begin{aligned}
v_i^*(\mu_i) &= \sup_{x_i \in \mathcal{X}_i^*(\mu_i)} \mathbb{E}_{\mu_i} v_i(x_i, \omega) \\
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Proof idea

- Upper bound immediate from Lemma
- Can approximate arbitrary $\mu \in \mathcal{M}(p_0)$ and give no info
- Augment types to convey any info about total state

What does the theorem buy us?

Multi-agent setting

- Agents' actions affect others' incentives
- Public information may be suboptimal
- i and j's information on ω (and x_k) has joint restrictions

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Theorem says you can

- Enrich the fundamental state to the total state
- Choose distribution (s.t. marginal on fundamentals)
- Design information one agent at a time

Example: team production

Moral hazard in teams

Special case in which

$$\begin{aligned} N &= \{1,2\} \\ \Omega &= \{1,2\} \\ p_0 &= \text{uniform} \\ X_i &= \mathbb{R}_+ \\ v_i(x_i,\omega) &= -x_i \end{aligned}$$

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$$\begin{aligned} N &= \{1,2\} \\ \Omega &= \{1,2\} \\ p_0 &= \text{uniform} \\ X_i &= \mathbb{R}_+ \\ v_i(x_i,\omega) &= -x_i \\ u_i(a,x_i,\omega) &= P_{a_1+a_2}x_i - a_i c_i(\omega) \end{aligned}$$

where

- $0 \leq P_0 < P_1 < P_2 \leq 1$
- $P_2 - P_1 > P_1 - P_0$
- $c_i : \Omega \to \mathbb{R}_{++}$

Firm's problem

Given belief μ_i with marginals μ_i^Π, μ_i^Ω:

- Let $\iota_i(\mu_i^\Pi)$ be i's expected marginal product if coworker
  - Works if ranked ahead of i
  - Shirks if ranked behind i (supermodularity)
- Then $\mathcal{X}_i^*(\mu_i) = \left\{x_i : \, x_i \iota_i(\mu_i^\Pi) > c_i(\mu_i^\Omega)\right\}$

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- Then $\mathcal{X}_i^*(\mu_i) = \left\{x_i : \ x_i \iota_i(\mu_i^\Pi) > c_i(\mu_i^\Omega)\right\}$
- Hence, $v_i^*(\mu_i) = -\frac{c_i(\mu_i^\Omega)}{\iota_i(\mu_i^\Pi)}$

$$\min_{\mu \in \mathcal{M}(p_0)} \sum_{i=1,2} \min_{\tau_i \in \Delta\Delta(\Pi \times \Omega)} \int \frac{c_i(\mu_i^\Omega)}{\iota_i(\mu_i^\Pi)} \, \mathrm{d}\tau_i(\mu_i)$$

$$\text{s.t.} \quad \int \mu_i \, \mathrm{d}\tau_i(\mu_i) = \mu$$

Explicitly characterized optimum

Some features

Propositions: For optimal (μ, τ_1, τ_2):

1. If $c_1(1) = c_1(2)$ and $c_2(1) = c_2(2)$, **no information**:
   neither agent learns anything about the total state

2. If $c_1(1) = c_2(2) > c_2(1) = c_1(2)$, **public information**:
   both agents learn the fundamental state

3. If $c_1(1) > c_2(2) = c_2(1) = c_1(2)$, **private information**:
   only agent 1 learns the fundamental state

Note: Higher-order belief features sometimes come for free

Concluding remarks

Joint incentive + information design under strategic uncertainty

Methodology (for binary actions) that

- Identifies the appropriate total state variable
- Reduces information design problem to be agent-by-agent

Flexible framework can be applied to varied settings

Thanks!