Cheap Talk with Transparent Motives

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November 2019

Consider an investor calling a broker to consult about an asset.

Broker knows how much the investor should hold.

Broker earns fees proportional to trade volume.

How much will the broker benefit from advising the investor?

Broker communicates with investor using cheap talk.

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Can’t we use Crawford & Sobel (1982)?

- Key feature: state matters for sender’s payoffs.
- Communication comes from single-crossing—i.e., separation motive.
- Here: broker’s information irrelevant for her payoffs.
- i.e. broker has state-independent preferences.

Broker communicates with investor using cheap talk.

Broker has state-independent preferences.

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Broker has state-independent preferences.

How about Chakraborty & Harbaugh (2010)?

- Study expert with state-independent preference.
- They look at multidimensional environment.
- Establish existence of influential communication.
- Idea: expert communicates by trading-off dimensions.
- Open question: What are the broker's benefits?

What do we do?

General cheap talk with state-independent sender preferences.

- Broker wants to generate trades.
- Salesperson wants to sell products.
- Think tank wants to implement agenda.
- Job candidate wants to get hired.

What do we do?

General cheap talk with state-independent sender preferences.

Main observation:
Sender gets credibility from garbling self-serving information.

Allows us to:

1. Characterize sender's equilibrium payoff set.
2. Cleanly compare cheap talk to commitment.
3. Solve wide range of applications.

Literature

Cheap talk:

- Crawford & Sobel (1982), Green & Stokey (1981, 2007), Battaglini (2002), Chakraborty & Harbaugh (2010), Margaria & Smolin (2018), Aumann & Hart (2003).

Persuasion / the belief-based approach:

- Kamenica & Gentzkow (2011), Rayo & Segal (2010), Brocas & Carrillo (2007), Aumann & Maschler (1966, 1995), Benoît & Dubra (2011).

"Constrained persuasion":

- Perez-Richet (2014), Salamanca (2017), Best & Quigley (2017), Lipnowski, Ravid, & Shishkin (2018).

Our model

$$\mu_0 \in \Delta\Theta, \qquad u_R : A \times \Theta \to \mathbb{R}, \qquad u_S : A \to \mathbb{R}.$$

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Sender-Receiver cheap talk game.

- S privately sees $\theta \in \Theta$.
- S sends R a message $m \in M$.
- R chooses $a \in A$.
- R gets payoff $u_R(a, \theta)$, S gets payoff $u_S(a)$.

We study (perfect Bayesian) equilibria.

$$\langle \sigma : \Theta \to \Delta M,\ \rho : M \to \Delta A,\ \beta : M \to \Delta\Theta \rangle.$$

Technical assumptions:

$$\begin{aligned} \Theta, A &: \quad \text{compact metrizable} \\ u_R, u_S &: \quad \text{continuous} \\ M &: \quad \text{rich (e.g. contains } \Delta A\text{)} \end{aligned}$$

Equilibrium outcomes

We study (perfect Bayesian) equilibria.

$$\langle \sigma : \Theta \to \Delta M,\ \rho : M \to \Delta A,\ \beta : M \to \Delta\Theta \rangle$$

Every equilibrium induces:

- Distribution of R's posterior beliefs, $p \in \Delta\Delta\Theta$.
- S's ex-ante payoff, $s \in \mathbb{R}$.

Call (p, s) an **equilibrium outcome**.

Our protagonists

$$V : \Delta\Theta \rightrightarrows \mathbb{R}, \qquad v : \Delta\Theta \to \mathbb{R}, \qquad \mathcal{I}(\mu_0) \subseteq \Delta\Delta\Theta.$$

S's value correspondence:

$$V(\mu) := \operatorname{co} u_S\left(\arg\max_{a\in A} \int_\Theta u_R(a,\cdot)\,\mathrm{d}\mu\right).$$

S's value function:

$$v(\mu) := \max V(\mu).$$

Information policies:

$$\mathcal{I}(\mu_0) := \left\{p \in \Delta\Delta\Theta : \int_{\Delta\Theta} \mu\,\mathrm{d}p(\mu) = \mu_0\right\}.$$

Consulting a broker

Investor is R, broker is S.

Investor's current position is $a_0 \in [0, 1]$.

Investor chooses new position, $a \in [0, 1]$, after consultation.

Broker knows ideal asset position, $\theta \in \{0, 1\}$.

Broker earns fees proportional to trade volume.

Consulting a broker

Simple example

$$A = [0, 1],$$

$$\Theta = \{0, 1\},$$

$$u_S(a) = |a - a_0|,$$

$$u_R(a, \theta) = -(a - \theta)^2,$$

$$\mu_0\{1\} = \frac{1}{3}.$$

Investor's best response

Given belief $\mu \in \Delta\Theta$, investor chooses:

$$a(\mu) = \mathbb{E}_{\mu}\theta = \mu\{1\}.$$

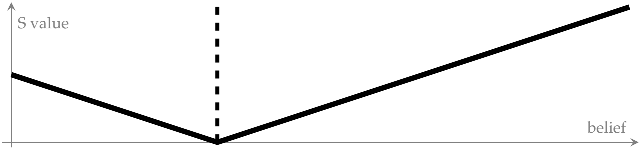
For simplicity, assume initial holdings are correct, i.e.

$$a_0 = \mathbb{E}_{\mu_0}\theta = \frac{1}{3}.$$

Broker's value function

S's value function:

$$v(\mu) = \left|\mu\{1\} - \frac{1}{3}\right|.$$



Equilibrium outcomes

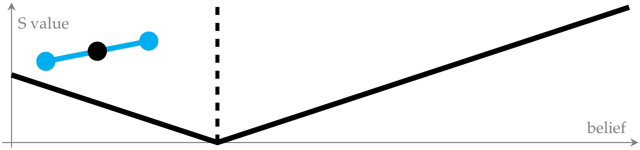
Characterizing equilibrium outcomes

The belief-based approach

In equilibrium, any message induces:

- A receiver *belief*.
- A receiver mixed action, which yields a sender *payoff*.

Think about equilibrium distributions of (*belief*, *payoff*) pairs.

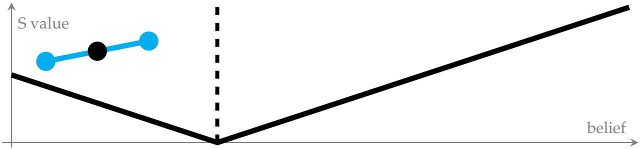


Characterizing equilibrium outcomes

The belief-based approach

Equilibrium distributions of (*belief*, *payoff*) pairs.

By Bayes, the $\Delta\Theta$ coordinate must average to the prior.

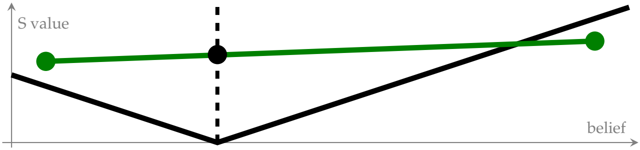


Characterizing equilibrium outcomes

The belief-based approach

Equilibrium distributions of (*belief*, *payoff*) pairs.

By R's best response, we have to live on the graph of V.

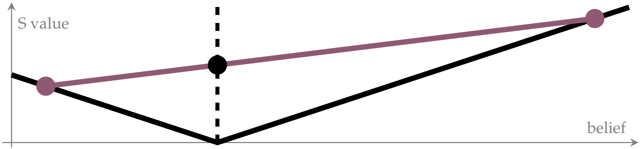


Characterizing equilibrium outcomes

The belief-based approach

Equilibrium distributions of (*belief*, *payoff*) pairs.

By S's best response, the payoff coordinate must be constant.

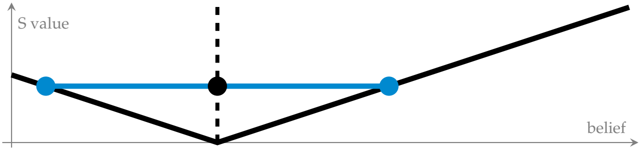


Characterizing equilibrium outcomes

The belief-based approach

Equilibrium distributions of (*belief*, *payoff*) pairs.

But then the *ex-ante* S payoff is equal to the *ex-post* payoff.

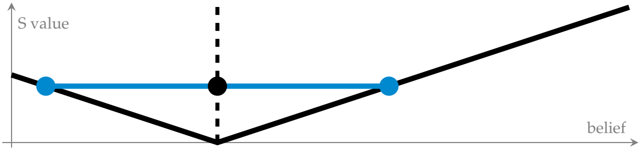


Characterizing equilibrium outcomes

The belief-based approach

Outcome (p, s) is an equilibrium outcome only if

- $p \in \mathcal{I}(\mu_0)$;
- $s \in \bigcap_{\mu \in \text{supp}(p)} V(\mu)$.



Characterizing equilibrium outcomes

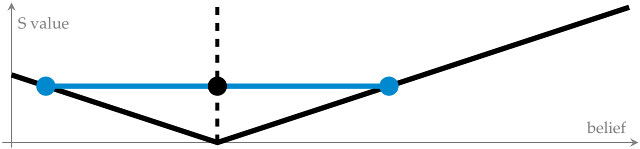
The belief-based approach

The Translation Lemma:

Outcome (p, s) is an equilibrium outcome if and only if:

- $p \in \mathcal{I}(\mu_0)$;
- $s \in \bigcap_{\mu \in \text{supp}(p)} V(\mu)$.

Essentially follows from Aumann & Hart (2003).

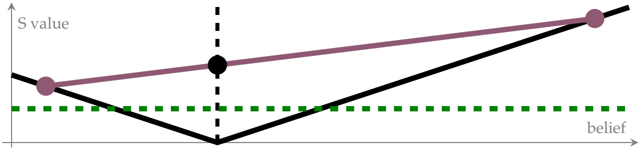


Securability

Definition:
Say $p \in \mathcal{I}(\mu_0)$ **secures** value $s \in \mathbb{R}$ if

$$p\{v \geq s\} = 1.$$

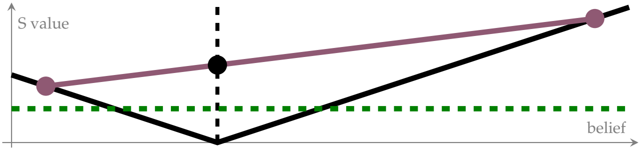
Say s is **securable** if there is some $p \in \mathcal{I}(\mu_0)$ that secures s.



Securability

Theorem 1: Let $s \geq v(\mu_0)$. The following are equivalent:

1. s is an equilibrium S payoff.
2. s is securable.



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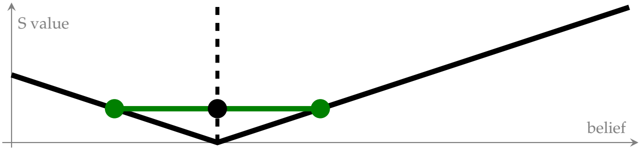
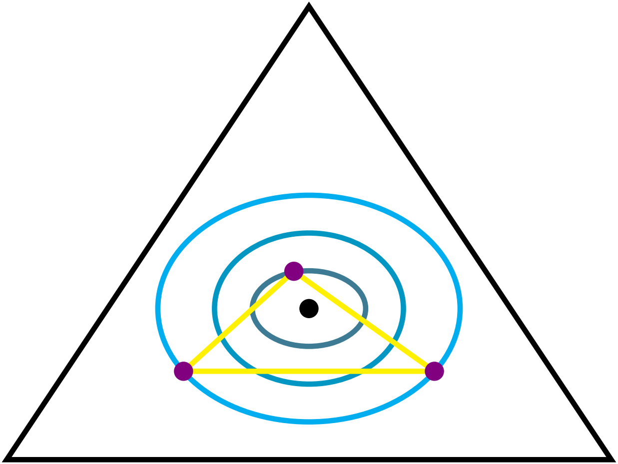


Securability

A constructive proof



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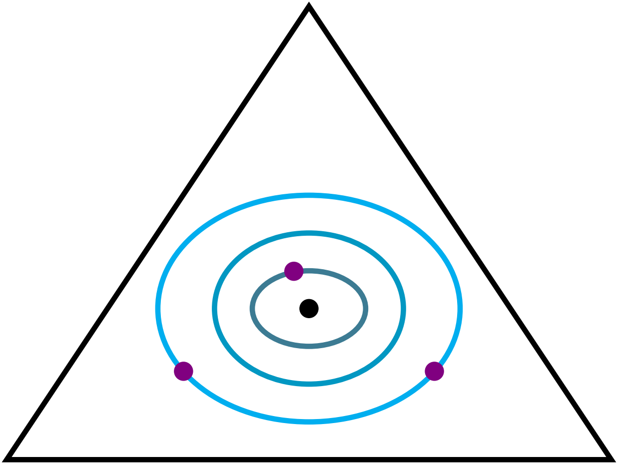

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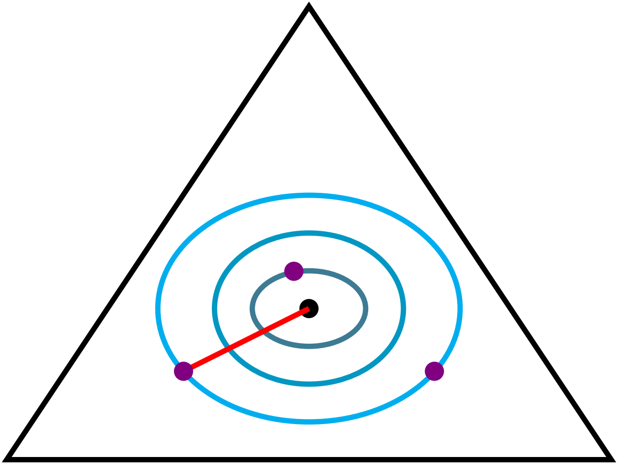

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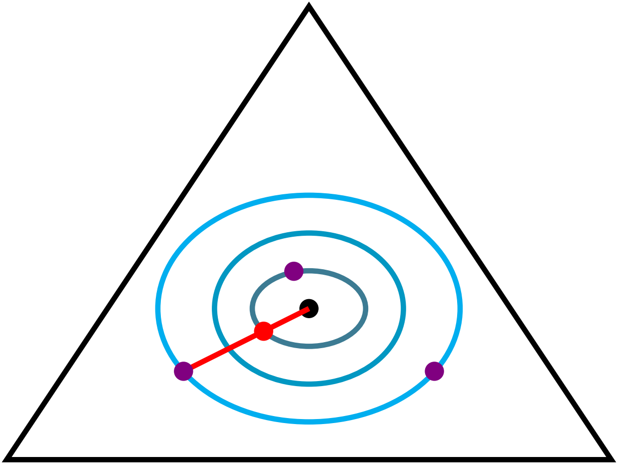

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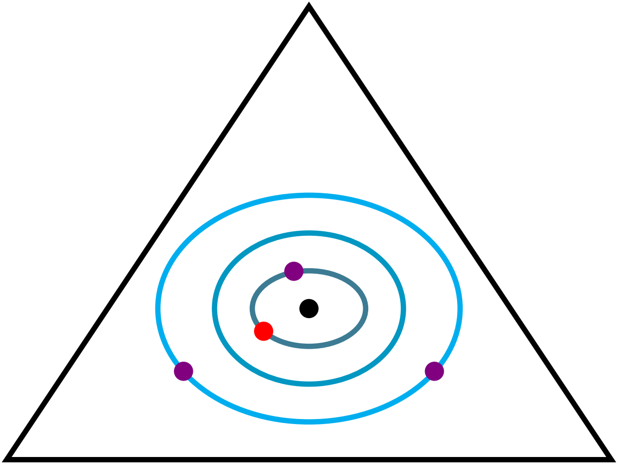

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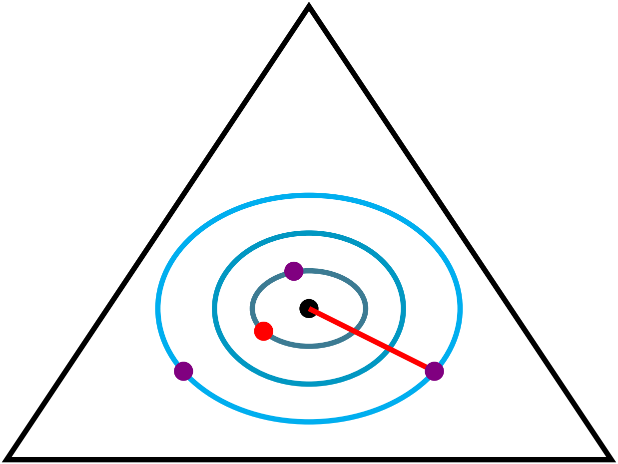

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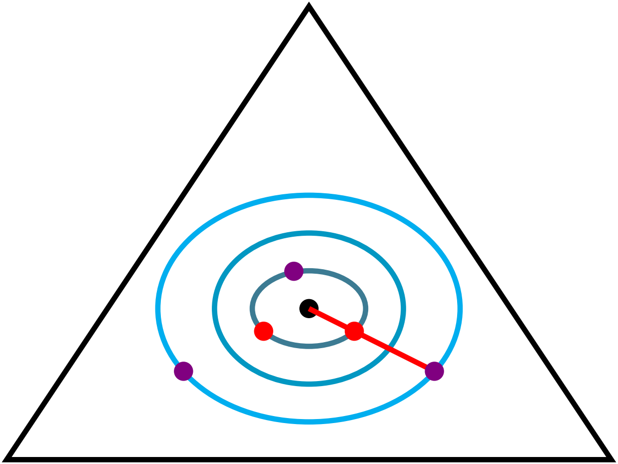

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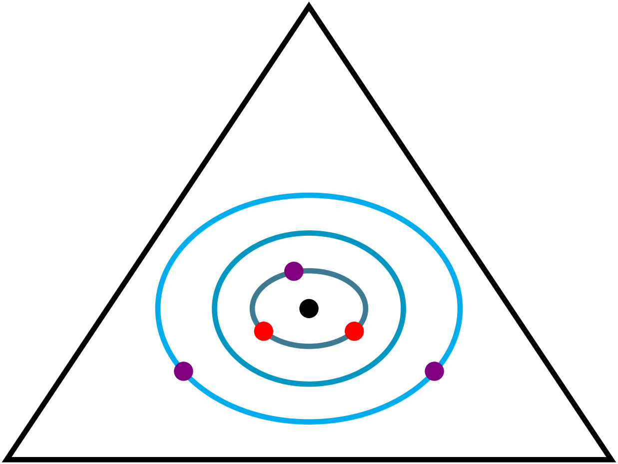
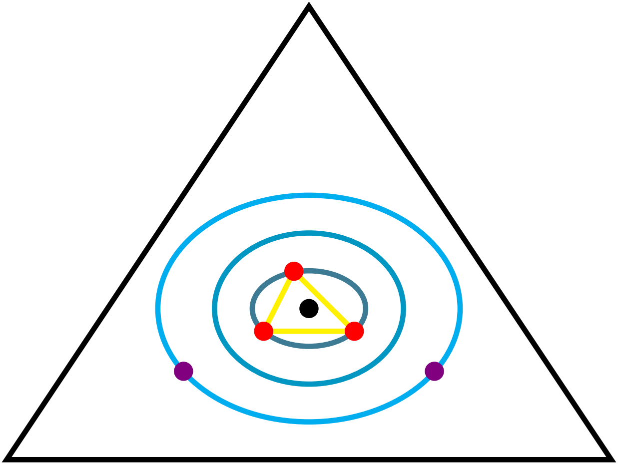


SECURABILITY

A CONSTRUCTIVE PROOF



Cheap talk

vs.

commitment

Commitment's value: ex-ante vs. ex-post

With commitment, S maximizes **ex-ante** value:

$$\hat{v}(\mu_0) = \max_{p \in \mathcal{I}(\mu_0)} \int_{\Delta\Theta} v \, \mathrm{d}p.$$

Commitment's value: ex-ante vs. ex-post

With commitment, S maximizes **ex-ante** value:

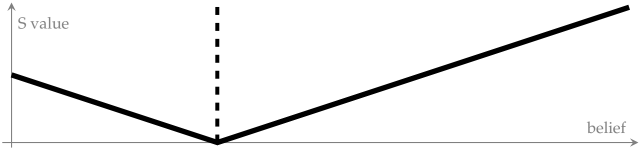
$$\hat{v}(\mu_0) = \max_{p \in \mathcal{I}(\mu_0)} \int_{\Delta\Theta} v \, \mathrm{d}p.$$

Without commitment, S maximizes lowest **ex-post** value:

$$\bar{v}(\mu_0) = \max_{p \in \mathcal{I}(\mu_0)} \inf \ v\,(\operatorname{supp} p).$$

Commitment's value: geometric perspective

Aumann & Maschler (1966), Kamenica & Gentzkow (2011):
The highest S payoff attainable with S commitment is the concave envelope of v, evaluated at the prior.



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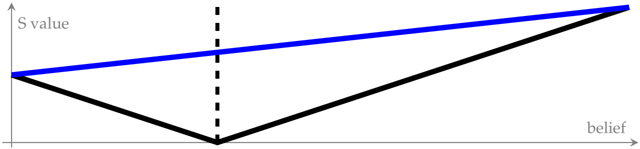
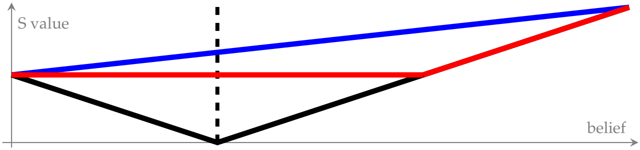
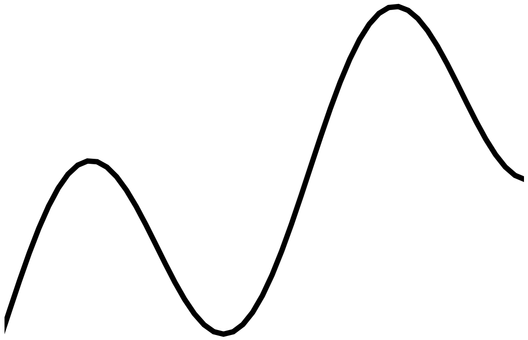


Commitment's value: geometric perspective

Theorem 2:
The highest S payoff attainable under cheap talk is the *quasi*concave envelope of v, evaluated at the prior.



Concave vs. Quasiconcave envelope



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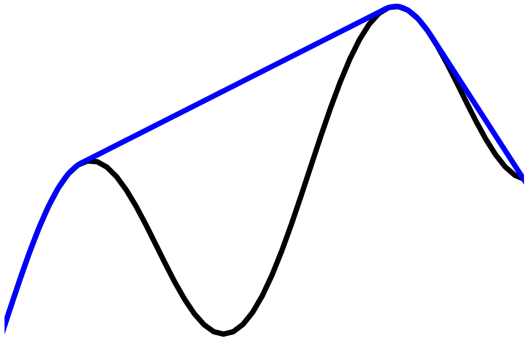

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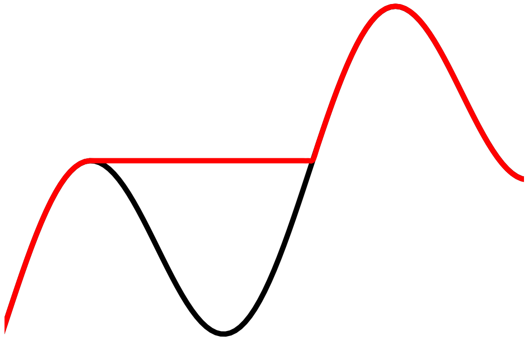

Quasiconcave envelope proof outline

Need to show four things:

1. $\bar{v}$ upper semicontinuous.

2. $\bar{v}$ majorizes v.

3. $\bar{v}$ is quasiconcave.

4. If f is u.s.c., quasiconcave, and above v, then $\bar{v} \leq f$.

Quasiconcave envelope proof outline

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Quasiconcave envelope proof outline

Need to show four things:

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2. $\bar{v}$ majorizes v. Follows from $\delta_{\mu_0} \in \mathcal{I}(\mu_0)$.
3. $\bar{v}$ is quasiconcave.
4. If f is u.s.c., quasiconcave, and above v, then $\bar{v} \leq f$.

$\bar{v}$ IS QUASICONCAVE

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$$\bar{v}\big((1-\lambda)\mu' + \lambda\mu''\big) = \max_{p \in \mathcal{I}((1-\lambda)\mu' + \lambda\mu'')} \inf v\big(\mathrm{supp}(p)\big)$$

$\bar{v}$ IS QUASICONCAVE

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&= \min\left\{\bar{v}(\mu'),\ \bar{v}(\mu'')\right\}
\end{aligned}
$$

Completing the proof

Fix quasiconcave, upper semicontinuous $f : \Delta\Theta \to \mathbb{R}$ above v.

Need to show: $\bar{v} \le f$.

Suppose $p \in \mathcal{I}(\mu_0)$ secures $\bar{v}(\mu_0)$.

Let $D = \text{supp } p$. Then,

$$\bar{v}(\mu_0) = \inf \ v(D)$$

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Commitment is usually valuable

Corollary: Suppose A, Θ are finite. Then, for Lebesgue-a.e. $\mu_0 \in \Delta\Theta$, one of the following holds:

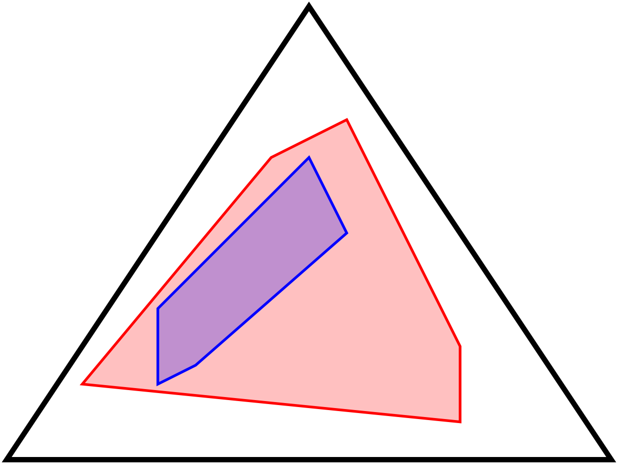
(i) Cheap talk yields S her first-best value,

$$\bar{v}(\mu_0) = \max v(\Delta\Theta).$$

(ii) Commitment is valuable,

$$\bar{v}(\mu_0) < \hat{v}(\mu_0).$$

Commitment is usually valuable



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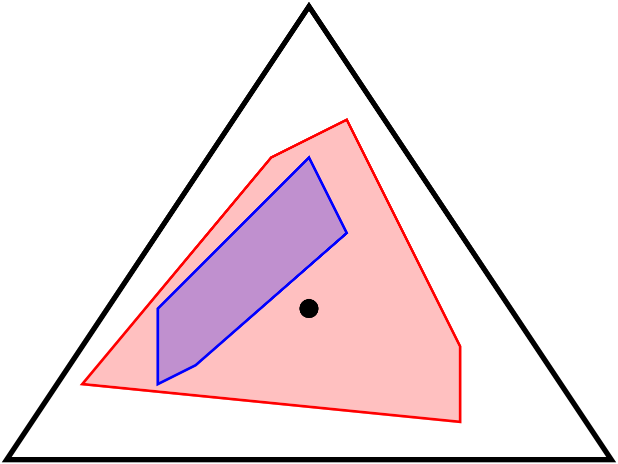

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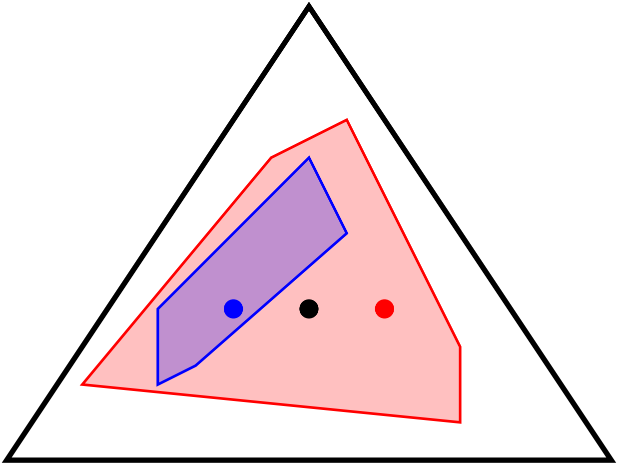

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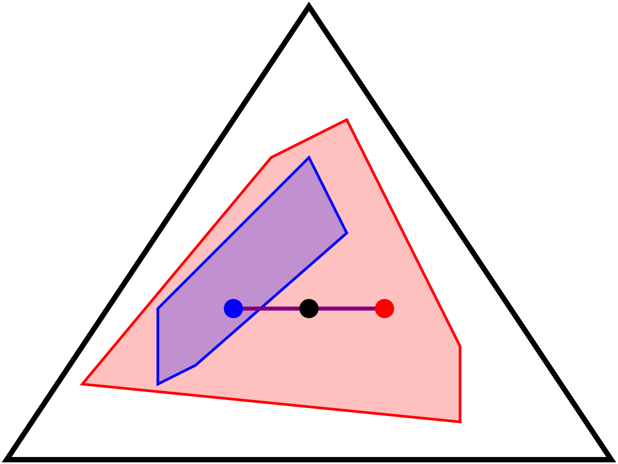

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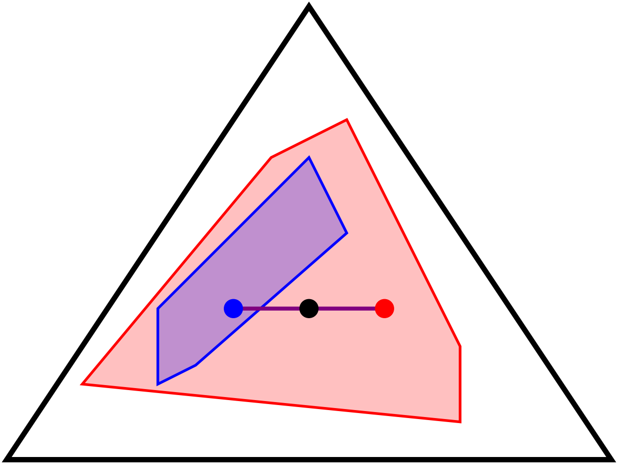
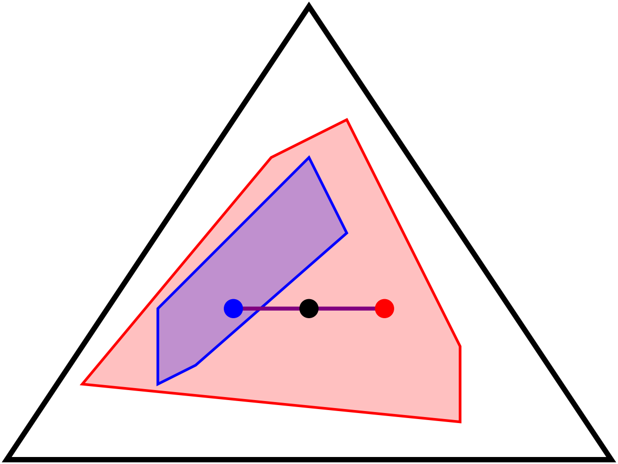


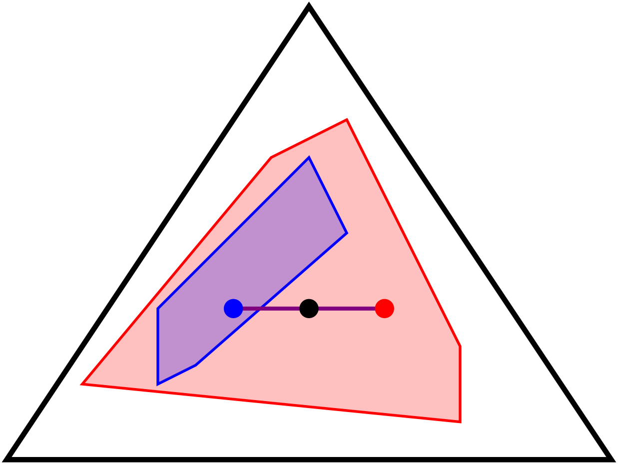
$\bar{v}(\mu_0) < \lambda\bar{v}(\mu) + (1 - \lambda)\bar{v}(\mu')$

Commitment is usually valuable



$$\bar{v}(\mu_0) < \lambda\bar{v}(\mu) + (1-\lambda)\bar{v}(\mu') \leq \lambda\hat{v}(\mu) + (1-\lambda)\hat{v}(\mu')$$

Commitment is usually valuable



$$\bar{v}(\mu_0) < \lambda\bar{v}(\mu) + (1-\lambda)\bar{v}(\mu') \leq \lambda\hat{v}(\mu) + (1-\lambda)\hat{v}(\mu') \leq \hat{v}(\mu_0).$$

Richer broker example

CONSULTING A BROKER

Investor (R) calls broker (S) to consult about an asset.

Investor's current position is $a_0 \in [0, 1]$.

Investor chooses new position, $a \in [0, 1]$, after consultation.

Broker knows ideal asset position, θ.

Broker earns fees proportional to trade volume.

A richer broker example

$$
\begin{aligned}
A &= [0,1], \\
\Theta &= [0,1], \\
\mu_0 &\in \Delta\Theta : \text{ atomless}, \\
u_S(a) &= \phi|a - a_0|, \ \phi > 0, \ a_0 = \mathbb{E}_{\mu_0}\theta, \\
u_R(a,\theta) &= -\tfrac{1}{2}(a-\theta)^2 - u_S(a).
\end{aligned}
$$

Broker's value function

For any $\mu \in \Delta\Theta$, the investor's best response is unique,

$$a^*(\mu) = \begin{cases} \mathbb{E}_\mu \theta + \phi & : \ \mathbb{E}_\mu \theta \le a_0 - \phi \\ a_0 & : \ \mathbb{E}_\mu \theta \in (a_0 - \phi, a_0 + \phi) \\ \mathbb{E}_\mu \theta - \phi & : \ \mathbb{E}_\mu \theta \ge a_0 + \phi. \end{cases}$$

So, the broker's value function is

$$v(\mu) = \phi \ \max\left\{\left|\mathbb{E}_\mu \theta - a_0\right| - \phi, \ 0\right\}.$$

v: CONVEX FUNCTION OF INVESTOR'S EXPECTATIONS

Broker's value function is $v(\mu) = h\left(\mathbb{E}_\mu \theta\right)$, where

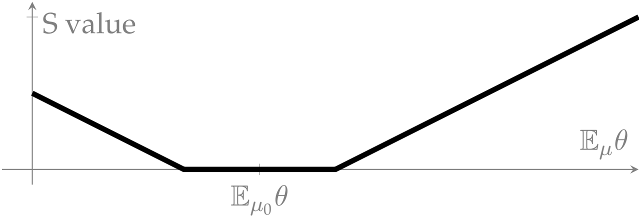
$$h(t) := \phi \ \max\left\{|t - a_0| - \phi, \ 0\right\}.$$

The broker's value: Slightly different graph

Broker's value function,

$$v(\mu) = h\left(\mathbb{E}_{\mu}\theta\right).$$

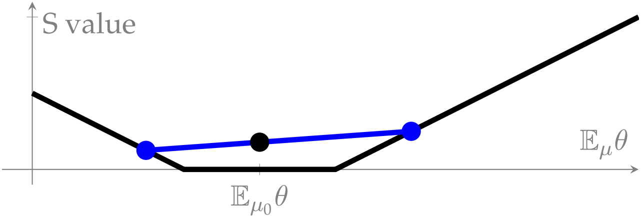
Can still draw graph, now with $\mathbb{E}_{\mu}\theta$ on horizontal axis.



THE BROKER'S VALUE: SLIGHTLY DIFFERENT GRAPH

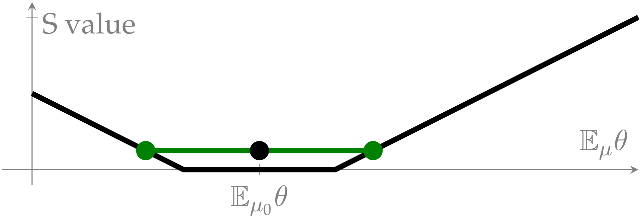
Equilibrium still induces distribution over points on graph.

What conditions must these distributions satisfy?



Binary policies are enough

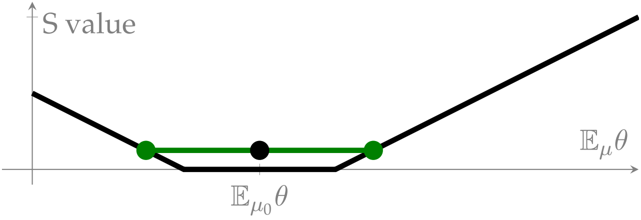
Being on $h(\cdot)$ and constant S payoff: still summarize incentives.



Binary policies are enough

Being on $h(\cdot)$ and constant S payoff: still summarize incentives.

Immediate: 2-message policies are sufficient.



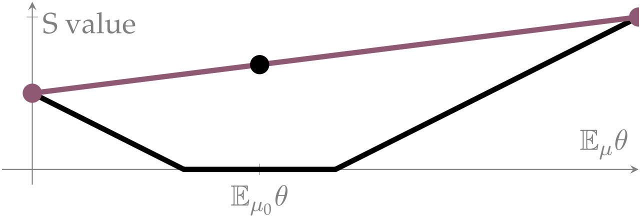
For Bayes: Martingale only necessary

Bayes-plausibility is more complicated.

Necessary: average ex-post mean equals ex-ante mean,

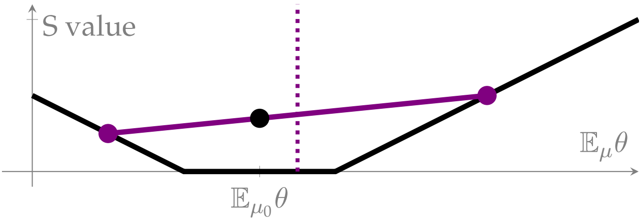
$$\int_{\Delta\Theta} \left[\mathbb{E}_{\mu}\theta\right] \, \mathrm{d}p(\mu) = \mathbb{E}_{\mu_0}\theta.$$

But mean being a martingale is not sufficient...



Cutoff policies: Simple feasible policies

Definition.
p is a θ^*-**cutoff policy** if it says whether θ is above or below θ^*.



Using securability in this setup

Claim:

The following are equivalent:

(i) s is attainable in equilibrium.

(ii) s is securable by a cutoff policy.

Proof sketch:

(i) implies (ii): Comes from convexity of h. ( )

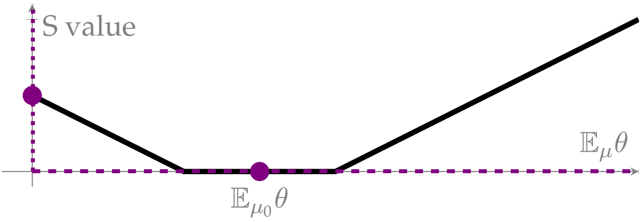
(ii) implies (i): From Theorem 1.

Securing values via cutoff policies

If cutoff is 0:

- Left message mean = 0,
- Right message mean = prior mean.

Secured value is 0.

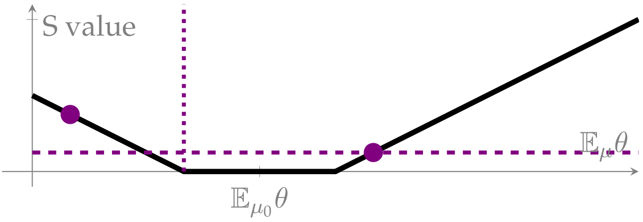


SECURING VALUES VIA CUTOFF POLICIES

As cutoff increases, both means increase. As such,

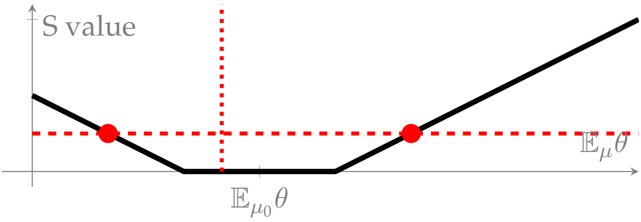
- continuation value from left message **decreases**,
- continuation value from right message **increases**.

At the beginning: secured value increases.



Securing values via cutoff policies

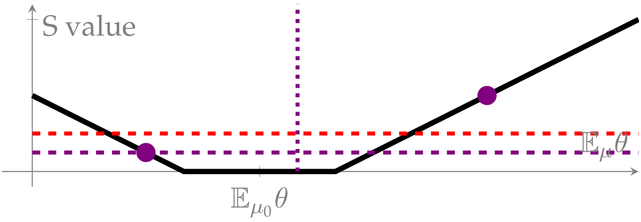
When the cutoff equals the median (in our example), both messages yield the same continuation value. Hence, median cutoff policy is sender IC.



Securing values via cutoff policies

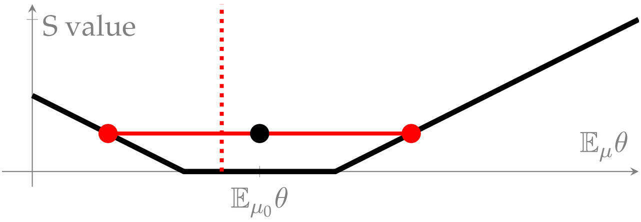
For cutoffs above median, secured value declines.

That is, median cutoff secures highest value.



Broker favorite equilibrium

$\Longrightarrow$ Median cutoff policy is broker favorite equilibrium.



Broker favorite equilibrium

Let $\theta_>$:= mean conditional on being **above** median.

Broker favorite equilibrium value:

$$h(\theta_>) = \phi \ \max\{\theta_> - a_0 - \phi, 0\}.$$

Broker's fee

- Broker's value is single peaked in the fee, ϕ.
- Optimal fee for broker: $\phi = \frac{1}{2}(\theta_> - a_0)$.
- Equilibrium information independent of fee.

Broker favorite equilibrium

Let $\theta_>$:= mean conditional on being **above** median.

Broker favorite equilibrium value:

$$h(\theta_>) = \phi \ \max\{\theta_> - a_0 - \phi, 0\}.$$

Market volatility (spread of prior)

- Mean-preserving spreads weakly increase broker's value.
- Broker only benefits from spread in *useful* information, i.e. in $\theta_> - a_0$.

Broker favorite also investor favorite

Boring algebra says investor's equilibrium value is

$$\tfrac{1}{2\phi}s^2 - \mathbb{V}_{\theta\sim\mu_0}[\theta],$$

where s is broker's value.

Broker favorite equilibrium is Pareto dominant.

Investor's payoffs in this equilibrium are:

$$\frac{1}{2}\left[\max\{\theta_{>} - a_0 - \phi,\ 0\}\right]^2 - \mathbb{V}_{\theta\sim\mu_0}[\theta].$$

Investor comparative statics

Investor's payoffs are:

$$\frac{1}{2}\left[\max\{\theta_> - a_0 - \phi,\ 0\}\right]^2 - \mathbb{V}_{\theta\sim\mu_0}[\theta].$$

Easy comparative statics:

- Investor is better off with lower ϕ.
- Volatility has an ambiguous effect.

The think tank

Reform selection

Think tank (S) advises lawmaker (R) on a reform.

Lawmaker can choose one of n proposals or status quo.

Proposal's values: unknown to lawmaker.

Think tank has expertise: knows each proposal's value.

Also has transparent motives: strict preference over proposals.

Reform selection

Think tank (S) advises lawmaker (R): status quo and n proposals.

$$A = \{0, 1, \ldots, n\}.$$

$$\Theta = [0,1]^n.$$

$$u_R(a,\theta) = \begin{cases} \theta_i - c & : a = i \in \{1, \ldots, n\}, \\ 0 & : a = 0. \end{cases}$$

$$u_S(a) = a \text{ (or something else increasing)}.$$

$$\mu_0 \in \Delta\Theta \text{ exchangeable without ties.}$$

Claim: The following are equivalent.

(i) $k \in \{1, \ldots, n\}$ is an equilibrium S value.

(ii) $\mathbb{E}\left[\max_{i \in \{k,\ldots,n\}} \theta_i - c\right] \geq 0$.

(iii) S can secure k by revealing the random variable

$$\mathbf{i}_k := \arg \max_{i \in \{k,\ldots,n\}} \theta_i.$$

Reform selection

Applying securability

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Proof:

(i) $\Longrightarrow$ (ii): Lawmaker ex-ante incentives. (▶)

(ii) $\Longrightarrow$ (iii): Exchangeability. (▶)

(iii) $\Longrightarrow$ (i): By Theorem 1.

Reform selection

Applying securability

Let $k^* := \max\left\{k \in \{1, \ldots, n\} : \mathbb{E}\left[\max_{i\in\{k,\ldots,n\}} \theta_i - c\right] \geq 0\right\}$.

The claim implies that

The S-preferred equilibrium value is k^*.

(If no such k^* exists, best is babbling.)

Reform selection

The following is an S favorite equilibrium.

- S names reform $\mathbf{i} \in \{k^*, \ldots, n\}$ given by:

$$\begin{cases} \mathbf{i}_{k^*} = \arg\max_{i \in \{k^*, \ldots, n\}} \theta_i & : \text{ w.p. } 1 - \epsilon, \\ \mathbf{i} \sim \text{uniform}\{k^*, \ldots, n\} & : \text{ w.p. } \epsilon. \end{cases}$$

where $\epsilon \in [0, 1)$ satisfies $(1 - \epsilon)\mathbb{E}\left[\max_{i \in \{k, \ldots, n\}} \theta_i\right] + \epsilon \mathbb{E}\left[\theta_j\right] = c$.

- R indifferent between i and outside option. Chooses:

$$\begin{cases} i & : \text{ w.p. } \frac{k^*}{i} \\ 0 & : \text{ w.p. } 1 - \frac{k^*}{i}. \end{cases}$$

Reform selection

A complete solution: the i.i.d. uniform case

If $(\theta_i)_i$ are i.i.d. uniform, then:

$$\mathbb{E}\left[\max_{i\in\{k,\dots,n\}} \theta_i\right] = \frac{1+n-k}{2+n-k}.$$

Hence, solution (k^*, ϵ) is given by (if $c > \frac{1}{2}$):

$$\begin{aligned} k^* &= \left\lfloor n - \frac{2c-1}{1-c} \right\rfloor, \\ \epsilon &= 2\left[(1-c) - \frac{2c-1}{n-k^*}\right]. \end{aligned}$$

Bounding commitment's value

Reform selection

Commitment's value is at least $\int_{\Delta\Theta} v \,\mathrm{d}p_{k^*} - k^*$.

BOUNDING COMMITMENT'S VALUE

REFORM SELECTION

Commitment's value is at least $\int_{\Delta\Theta} v \, \mathrm{d}p_{k^*} - k^*$.

$$\tfrac{1}{2}(n - k^*).$$

Or, more generally,

$$\sum_{i=k^*}^{n} \tfrac{u_S(i) - u_S(k^*)}{n - k^* + 1}.$$

Note: solving the commitment case is difficult.

Connections to the literature

Informative communication

Battaglini (2002) and Chakraborty & Harbaugh (2007):

- *Trading off issues* can credibly convey information.

Chakraborty & Harbaugh (2010) showed this idea yields influential communication in a special case of our setting.

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Chakraborty & Harbaugh (2010) showed this idea yields influential communication in a special case of our setting.

Main idea of Chakraborty & Harbaugh (2010):

- State a multidimensional vector, prior admitting a density.
- S's payoff a continuous function of R's expectation.

$\implies$ Fixed point theorem applies.

Proposition:

Let $T : \Delta\Theta \to \mathcal{X}$ be continuous, where $\mathcal{X}$ is locally convex. If $T(\Theta)$ is noncollinear, $E_\mu T$ is non-constant for some equilibrium.

Long cheap talk

Well-known that multiple rounds expand outcome set (Forges 1990, Krishna & Morgan 2004, Aumann & Hart 2003).

Aumann & Hart (2003) characterize the outcome set from long cheap talk (i.e. ∞ many rounds):

- Characterization in terms of di/bi-convex hull.
- AH'86 $\rightsquigarrow$ equivalent to separation by bi-convex functions.
- Securability delivers a separating function for S's payoffs.

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- Securability delivers a separating function for S's payoffs.

Proposition: Allowing for long cheap talk does not change S's equilibrium payoff set.

Note: Long cheap talk can still help the receiver.

What we've seen...

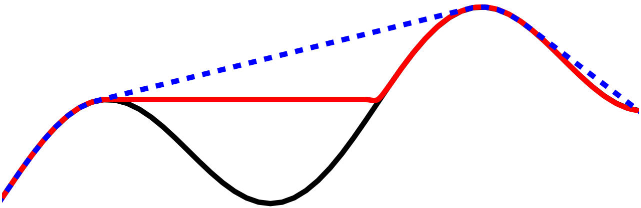
We studied general **cheap talk with transparent motives**.

Key observation: **securability**.

Credibility gained by degrading self-serving information.

- Clean comparison to the commitment case.
- Can explicitly solve wide range of applications.

Thanks!



Literature

Cheap talk:

- Crawford & Sobel (1982), Green & Stokey (2007), Battaglini (2002), Chakraborty & Harbaugh (2010), Margaria & Smolin (2018), Aumann & Hart (2003).

Persuasion / the belief-based approach:

- Kamenica & Gentzkow (2011), Rayo & Segal (2010), Brocas & Carrillo (2007), Aumann & Maschler (1966), Benoît & Dubra (2011).

“Constrained persuasion”:

- Perez-Richet (2014), Salamanca (2017), Best & Quigley (2017), Lipnowski, Ravid, & Shishkin (2018).

Definition of equilibrium

An **equilibrium** is a triple of measurable maps $\langle \sigma : \Theta \to \Delta M,\ \rho : M \to \Delta A,\ \beta : M \to \Delta\Theta \rangle$ such that:

- $\sigma\left(\arg\max_{m \in M} \int_\Theta u_S \,\mathrm{d}\rho(\cdot|m) \,\middle|\, \theta\right) = 1 \ \forall \theta \in \Theta.$
- $\rho\left(\arg\max_{a \in A} \int_\Theta u_R(a,\cdot) \,\mathrm{d}\beta(\cdot|m) \,\middle|\, m\right) = 1 \ \forall m \in M.$
- $\int_{\hat{\Theta}} \sigma(\hat{M}|\cdot) \,\mathrm{d}\mu_0 = \int_\Theta \int_{\hat{M}} \beta(\hat{\Theta}|\cdot) \,\mathrm{d}\sigma(\cdot|\theta) \,\mathrm{d}\mu_0(\theta)$
  $\forall$ Borel $\hat{\Theta} \subseteq \Theta$ and $\hat{M} \subseteq M$.

Reform selection

(i) implies (ii)

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&= \int_\Theta \left[\max_{i \in \{k,\dots,n\}} \theta_i - c \right] \mathrm{d}\mu_0(\theta) = \mathbb{E}\left[\max_{i \in \{k,\dots,n\}} \theta_i - c \right].
\end{aligned}$$

Reform selection

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Suppose (ii) holds – i.e., $\mathbb{E}\left[\max_{i\in\{k,\dots,n\}} \theta_i - c\right] \geq 0$.

Will show: Telling R to choose $\mathbf{i}_k$ is an IC recommendation to R.

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2. $\mathbb{E}[\theta_i|\mathbf{i}_k = i] \geq \mathbb{E}[\theta_i] = \mathbb{E}[\theta_j|\mathbf{i}_k = i]$ for all $j \in \{1, \ldots, k-1\}$.

Hence, choosing i_k^* is better for R than any $j \in \{0, \ldots, k-1\}$.

So: Telling R to choose i_k^* is IC for R, hence secures k.

▸ ...

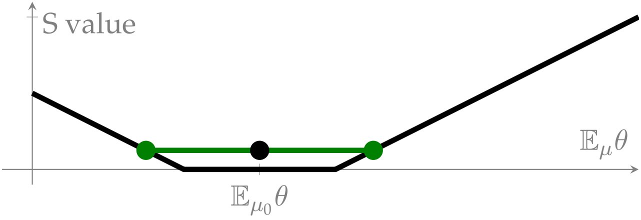
WHY CUTOFF POLICIES?

Lemma:

Let $p \in \mathcal{I}(\mu_0)$ be a binary policy inducing means $\theta_L < \theta_R$.

Then, $\exists$cutoff policy, $p^* \in \mathcal{I}(\mu_0)$, inducing means $\theta_L^* < \theta_R^*$ s.t.

$$\theta_L^* \leq \theta_L < \theta_R \leq \theta_R^*.$$



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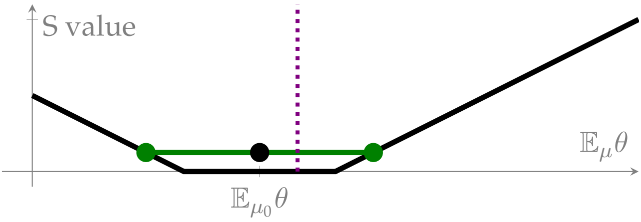

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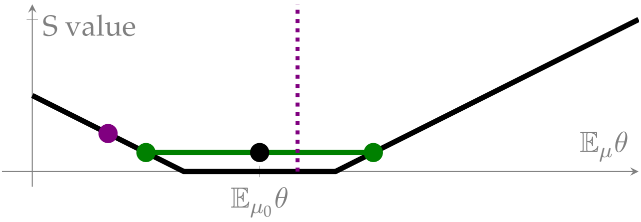

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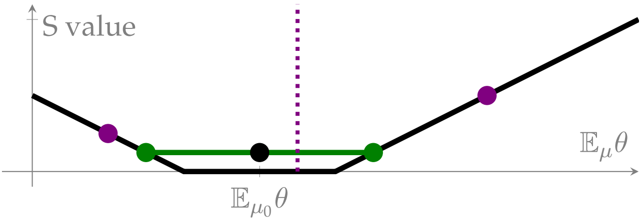
Then, $\exists$cutoff policy, $p^* \in \mathcal{I}(\mu_0)$, inducing means $\theta_L^* < \theta_R^*$ s.t.

$$\theta_L^* \leq \theta_L < \theta_R \leq \theta_R^*.$$



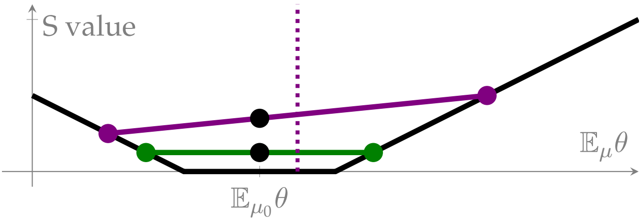
Why cutoff policies?

Lemma:

Let $p \in \mathcal{I}(\mu_0)$ be a binary policy inducing means $\theta_L < \theta_R$.

Then, $\exists$cutoff policy, $p^* \in \mathcal{I}(\mu_0)$, inducing means $\theta_L^* < \theta_R^*$ s.t.

$$\theta_L^* \leq \theta_L < \theta_R \leq \theta_R^*.$$



Why cutoff policies?

Since h convex & minimized at $a_0 = \int_\Theta \theta \, \mathrm{d}\mu_0(\theta)$, infer that

$$h(\theta_L^*) \geq h(\theta_L) \text{ and } h(\theta_R^*) \geq h(\theta_R).$$

Hence: p^* secures weakly higher value than p.